DOCUMENT RESUME

ED 139 662 SE 022 555

AUTHOR Graeber, Anna O.; Wallace, Lisa
TITLE Identification of Systematic Errors: Final Report.
INSTITUTION Research for Better Schools, Inc., Philadelphia, Pa.
SPONS AGENCY National Inst. of Education (DHEW), Washington, D.C.
PUB DATE 9 Feb 77
NOTE 80p.; For related document, see SE 022 556; Contains occasional light and broken type

EDRS PRICE MF-$0.83 HC-$4.67 Plus Postage.
DESCRIPTORS Basic Skills; Calculation; *Educational Diagnosis; *Educational Research; *Elementary School Mathematics; Elementary Secondary Education; *Error Patterns; Instruction; Mathematics Education
IDENTIFIERS *Number Operations; Research Reports

ABSTRACT

This study was designed to provide information about the types and frequency of systematic errors made by elementary school children in working mathematics examples. Student work samples on addition, subtraction, and multiplication were collected from two elementary schools and errors were classified and analyzed. Details are reported for errors made for each of 5 individual skills for addition, 5 for subtraction, and 2 for multiplication. Appendices include statements of objectives for analyzed skills, a table showing the distribution of pretests by skill and grade level, a sample data sheet, and a vocabulary list for systematic error descriptions. (DT)

****
* Documents acquired by ERIC include many informal unpublished *
* materials not available from other sources. ERIC makes every effort *
* to obtain the best copy available. Nevertheless, items of marginal *
* reproducibility are often encountered and this affects the quality *
* of the microfiche and hardcopy reproductions ERIC makes available *
* via the ERIC Document Reproduction Service (EDRS). EDRS is not *
* responsible for the quality of the original document. Reproductions *
* supplied by EDRS are the best that can be made from the original. *
****

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

SE

ED139662

IDENTIFICATION OF SYSTEMATIC ERRORS:

FINAL REPORT

Submitted to

National Institute of Education

by
Anna O. Graeber
and
Lisa Wallace

Research and Development Division
Research for Better Schools, Inc.
Suite 1700/1700 Market Street
Philadelphia, Pennsylvania 19103

February 9, 1977

2

SE 022 555

TABLE OF CONTENTS

	Page
Identification of Systematic Errors	1
Addition Skills	8
Subtraction Skills	25
Multiplication Skills	49
Disscussion and Implications	62
Appendix A: IPI Objective Statements for Analyzed Skills and the Related Table in Cox's Study	66
Appendix B: Distribution of Pretests by Skill and Grade Level	69
Appendix C: Data Sheet and Vocabulary List for Systematic Error Descriptions	71

IDENTIFICATION OF SYSTEMATIC ERRORS:

FINAL REPORT

One aspect of mathematics teaching that theorists, researchers, and practitioners have identified as important is that of diagnosis and prescription. The workscope on this topic focused on mathematics diagnostic-prescriptive classroom activities of K-8 teachers. Plans outlined in the original proposal to NIE called for knowledge to be collected from three sources: (1) a literature search, (2) a classroom survey, and (3) a small scale study of systematic errors. The original plan for the work unit also included an activity that would have explored methods for utilizing the recommendations of local teachers, teacher trainers, mathematicians and mathematics coordinators for the improvement of teachers' diagnostic-prescriptive skills. The identification of systematic errors was designed to provide information about the types and frequency of students' systematic errors (errors made over and over by adherence to some faulty "rule").

The present effort was not begun without knowledge of other work in the area nor without reason to believe that a teacher's ability to diagnose and differentiate between types of errors is important. Several sources documented in the final report of the literature search for this work unit[1] either describe systematic (or patterned) errors or offer rationale for the need for teachers to diagnose. Only a few of those are mentioned here.

---

[1]Anna O. Graeber, "Math Diagnosis and Prescription Literature Search," Final Report, (Philadelphia, Penna.: Research for Better Schools, Inc., 1976).

A report by Linda S. Cox[2] describes and classifies systematic errors made among both normal and handicapped student populations. The fact that prescriptions written by teachers using IPI Mathematics did not reflect the specific strengths or weaknesses the students demonstrated on their pretests is documented by Bolvin[3] and Graeber.[4] Other reports[5] indicate that this phenomenon is not limited to teachers using IPI. Yet, there is today a demand for teachers skilled in diagnosis. The present study was designed to provide more information about the types and frequency of systematic errors. The work builds on that of Cox. However, where Cox used samples of student work taken after instruction, the IPI pretests used in the present study sample students' work prior to instruction in the specific skill.

The work on analysis of systematic errors was initiated in September, 1976. Student work samples, in the form of completed IPI Pretests, had been collected from two schools during the 1974 - 1975 school year.

---

[2]Linda S. Cox, Analysis, Classification, and Frequency of Systematic Error Computational Patterns in the Addition, Subtraction, Multiplication, and Division Vertical Algorithms for Grades 2-6 and Special Education Classes, Final report, U.S. Department of Health, Education & Welfare, National Institute of Education Contract #SE018001 (Kansas City: University of Kansas Medical Center, June, 1974.)

[3]John O. Bolvin, "Evaluating Teacher Functions," Working Paper 17, (Pittsburgh, Penna.: Learning Research and Development Center, 1967), pp. 8-10.

[4]Anna O. Graeber, "Diagnostic-Test-Based Prescriptions in Individually Prescribed Instruction in Mathematics" (unpublished doctoral dissertation, Teachers College, Columbia University, New York, 1974).

[5]Jon M. Engelhardt, "Remediation of Learning Difficulties in School Mathematics" (paper presented at The First National Conference on Remedial Mathematics, Kent State University, Kent, Ohio, 1974); and Irons, Jerry, "The Need for Packaged Material and a Delivery System," From Proceedings of the Third National Conference on Remedial Mathematics, (Kent, Ohio: Kent State University, 1976).

In IPI Mathematics, instruction for specific skills does not begin until the student's subject matter competencies are determined. Placement tests are taken by students new to the IPI Mathematics Program or, in some schools, by all pupils at the beginning of a new school year. The tests are used to identify the units of work appropriate to the individual student's ability. The placement test for a given level tests only those skills in each unit that are the critical or terminal skills. Scores on these criterion referenced tests specify the level at which the student should begin work in each of ten areas. Once the placement testing procedure has been used to identify the unit in which a student will begin working, the unit pretest is given to determine those skills in the unit the student has not mastered and those he/she has matered. Results on the criterion referenced pretest identify the specific skills in which the pupil needs instruction. Each skill in a unit is tested on the pretest. The length of the pretest varies from skill to skill, but there are usually between 5 and 10 items. Completed IPI pretests for levels C, D, and E in the areas of Addition/Subtraction and Multiplication were utilized in the present study.

The skills for which analysis of systematic errors was carried out involved algorithms for the addition, subtraction, and multiplication of whole numbers that are beyond recall of basic facts but are not so complex or lengthy that the computation is almost always likely to be performed on a calculator. Analysis for twelve skills, five in addition, five in subtraction, and two in multiplication, are reported below. The IPI skill number and a statement of each objective may be found in Appendix A. The chart found in Appendix A also indicates which of the Tables in

Cox's[6] study contains data for the skill (as defined by Cox) that most closely matches the specified IPI skill. Original plans (assuming continued funding) called for the analysis of four additional multiplication skills and three division skills.

The two schools from which completed pretests had been collected were both very familiar with the IPI Math Program. One of the schools was in its sixth year of using IPI; the other school was in its fifth year of using IPI. RBS had maintained contact with both schools and the implementation of IPI Mathematics in both schools was satisfactory. Testing, for example, was carried out according to the developers' intents. One of the schools is located in a northeastern Pennsylvania city of approximately 58,000 inhabitants. The school had approximately 650 students in grades K-6 at the time the data were collected. Pretests from all 1st to 6th grade classes, with the exception of the two special education classes, were collected by RBS. Several other individualized programs were in operation in the school. Basically, the school was organized in self-contained classrooms. The second school, with a K-4 population of approxiamately 400, is located in an east central New Jersey suburban district. Six of the twelve first, second, third, and fourth grade teachers cooperated by sending all completed pretests to RBS. This school followed the Individually Guided Education (IGE) scheme of organization and included several classrooms with multi-age grouping. Appendix B gives the distribution of the analyzed tests by grade level.

Although student pretests had been scored at the schools, prior to submission to RBS, pretests were first checked against the published answer

---

[6]Cox, op. cit.

keys to assure accuracy and pinpoint errors. For each pretest a data sheet (see Appendix C) was completed. Where systematic errors occurred, the researcher first checked the error against those noted by Cox.[7] If the systematic error had been described by Cox, a code for the error was recorded on the data sheet. If the systematic error was not reported by Cox, a description involving terms from a specified vocabulary list (see Appendix C) was written and coded. For purposes of consistency, all systematic errors including those identified by Cox were eventually described in terms of the specified vocabulary and items from each pretest were used to develop illustrative examples.

Errors were not classified as systematic errors unless three or more of the test items evidenced the same error pattern. This definition is suggested in the writings of Grossnickle[8] and is also the one Cox utilized.

Naturally, not all of the observed errors were systematic ones. Each pretest was, however, tabulated in at least one of the other categories that was utilized. Some of these categories resulted from the definition of systematic errors, for example, categories for pretests with three or more errors for which no pattern could be detected. Other categories resulted from management techniques inherent in the IPI Mathematics Program. Before defining each category used for tabulation, some information about IPI will clarify the choice of some of the categories.

---

[7] Ibid.

[8] F.E. Grossnickle, "Reliability of Diagnosis of Certain Types of Errors in Long Division With a One-Figure Divisor," Journal of Experimental Education, IV (September, 1935), pp. 7-16.

In IPI, the pretest for a skill generally contains between five and twelve items. A certain number of these items must be answered correctly, if the student is to be considered as having "mastered" the pretest.[9] Of the twelve different skills analyzed, nine of the pretests had 9 items, one pretest had 8 items, and two pretests each had 6 items. Although criteria for skill mastery vary from skill to skill, for each of the 12 skills analyzed one or no errors on the pretest would lead to skill mastery. Two or more errors placed the student in the non-mastery group. Thus, in the categories used to report data for the twelve skills, the percent of students who exhibited one or no errors is the percent of students who mastered the skill on the pretest.

The definition of non-systematic error categories used in tabulation follows.

Fact Errors. Only those tests where students made fact errors in three or more items were tallied in this category. The category was included since the comparison between papers with three or more items missed due to fact errors and the percent of papers with three or more items missed as a result of systematic errors may be of interest. Also Lankford's[10] work suggests that poor computers have a limited stock of

---

[9]Students who "master" a skill in the pretest are not assigned in-instruction in that skill unless they fail to maintain their "mastery" status on the unit posttest.

[10]Francis G. Lankford, Jr., Some Computational Strategies of Seventh Grade Pupils, Final report, U.S. Department of Health, Education, and Welfare, Office of Education National Center for Educational Research and Development (Regional Research Program) and The Center for Advanced Study, Project Number 2-C-013, Grant Number OEG-3-0035 (Charlottesville, Virginia: The University of Virginia, October, 1972),

basic facts and also have difficulty remembering conventional algorithms. The number of students who are counted in both the fact error category and the systematic error category will be of interest.

Three or more random errors. All pretests with errors in three or more items that were neither the result of only fact errors nor (in as far as could be determined) systematic errors were tallied in this category. Again, the number of pretests counted in both this and the previous category would be of interest in characterizing the poor computer group.

Some of the individual errors encountered on tests tabulated in this category could be described by one of the identified systematic errors. However, the type of error varied from problem to problem so that the consistency necessary for the tabulation as a systematic error was lacking.

Two errors. Pretests with only two items incorrect were tallied in this category. Again, it was obvious to the researcher that many of the errors made on tests tabulated in this category could be described by the errors noted among the systematic errors. Again, however, the consistency was missing. It should also be remembered that some of the tests tabulated in this category had two items incorrect because of fact errors. For all skills under consideration, two incorrect items led to a non-mastery classification in the IPI program.

One or no error. Pretests containing one or no incorrect item were tallied in this category. For all skills under consideration, a student incorrectly answering only one or no items would be considered to have achieved "mastery" of the skill.

Did not attempt pretest. The recommended procedure for students who at the time of the pretest feel that they do not know enough about

the skill to attempt any work is for the student to indicate this on the test and to proceed to the next test. The number of tests on which this occurred is noted by the number tallied in the "Did not attempt pretest" category.

All of the above categories were utilized for each of the twelve skills analyzed. In the case of the two multiplication skills, a seventh category, described in the section below where those specific skills are discussed, was utilized. The data on each of the individual skills follows below in the order addition, subtraction, multiplication.

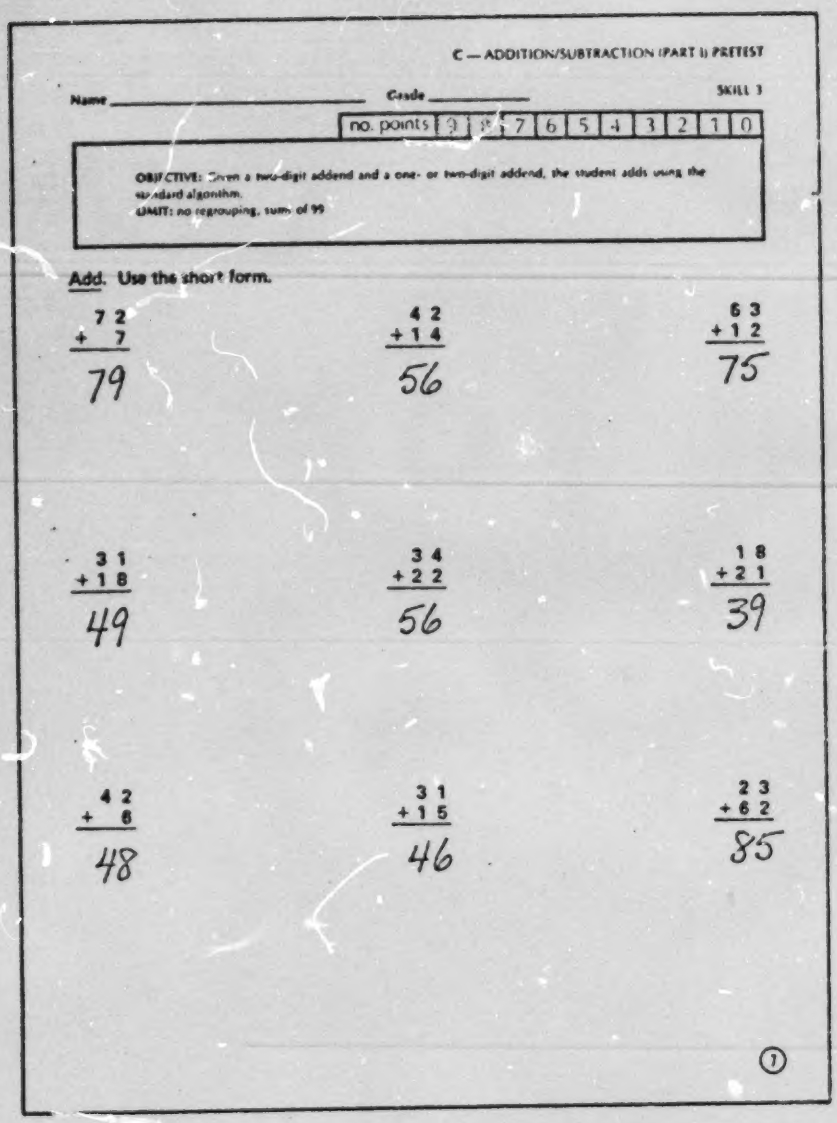
Addition Skills

Pretests for five addition skills were analyzed. The skills ranged from addition of two two-digit numbers with no regrouping to addition of two three-digit numbers with regrouping in the tens' and ones' columns. Figures 1-5, which follow, show the pretests with the correct answer for each item. Tables 1-5, below, include descriptions of each of the systematic errors noted and the number and percent of tests falling in each of the six previously defined categories. A total of 409 tests were analyzed: 71 for C-Addition/Subtraction-3, 69 for C-Addition/Subtraction-4, 70 for C-Addition/Subtraction-7, 100 for D-Addition/Subtraction-3, and 99 for D-Addition/Subtraction-4.

C-Addition/Subtraction-3. Of the 71 tests examined, 8.4% contained one of the three distinct systematic errors encountered. Only one test was tallied in both a systematic error category and in the fact error category.

Cox, in her post-instructional survey, found a systematic error rate of less than 1% on this skill among normal student populations.[11]

[11]Cox, op. cit., p. 31.



C — ADDITION/SUBTRACTION (PART I) PRETEST

Name _____ Grade _____ SKILL 3

no. points	9	8	7	6	5	4	3	2	1	0

OBJECTIVE: Given a two-digit addend and a one- or two-digit addend, the student adds using the standard algorithm.
LIMIT: no regrouping, sum of 99

Add. Use the short form.

72 + 7 = 79	42 + 14 = 56	63 + 12 = 75
31 + 18 = 49	34 + 22 = 56	18 + 21 = 39
42 + 6 = 48	31 + 15 = 46	23 + 62 = 85

1

Figure 1. C-Addition/Subtraction-3 Pretest with Answers

Table 1. C-Addition/Subtraction-3 Pretest Errors

Error				Frequency
Examples			Description	Number (percent)
72 + 7 = 16	42 + 14 = 11	63 + 12 = 12	The digits in the top addend and in the bottom addend are added separately disregarding place-values. (72 + 7 = 7 + 2 + 7 = 16)*	3 (4.2)
72 + 7 = 77	42 + 14 = 52	63 + 12 = 72	The tens' column is added correctly but no addition is performed in the ones' column. The one's digit of one of the addends is placed in the one's digit of the answer.	2 (2.8)
72 + 7 = 9	42 + 14 = 6	63 + 12 = 9	The digits in the top addend are added separately disregarding place-values. The bottom addend is ignored. (72 + 7 = 7 + 2 = 9)	1 (1.4)
			Fact errors	2 (2.8)
			Three or more random errors	5 (7.0)
			Two errors	5 (7.0)
			One or no errors	54 (76.1)
			Did not attempt pretest	0 (0.0)
				n = 71[1]

[1] The sum of the numbers is greater than 71, and the percents total more than 100, because one or more students were counted in more than one category.

* Also noted by Linda S. Cox, (1974).

Although Cox did not find any of the errors described in Table 1 of the present study in the tests she examined for the same skill (her Addition, Level 3), one of the systematic errors described in Table 1 is reported in Cox for her Addition, Level 4.[12] Neither of the two errors Cox reported for Addition, Level 3 was found among the C-Addition/Subtraction-3 tests included in the present study.[13]

C-Addition/Subtraction-4. On the C-Addition/Subtraction-4 skill that involves regrouping, more systematic errors were documented and, as Table 2 shows, a greater percent of papers, 21.7%, evidenced systematic errors. None of the tests evidenced two systematic errors or a systematic error and three or more fact errors. The number of tests with systematic errors, 15, is greater than the number of tests with three or more fact errors, 3, and the number of tests with three or more random errors, 2. The majority of tests that included systematic errors involved faulty regrouping.

The C-Addition/Subtraction-4 skill in IPI is equivalent to Cox's Addition, Level 2 skill. Cox found systematic errors on 11% of the tests from her second, third, and fourth grade normal population.[14] At the second grade level, the grade level of most of the students whose tests were used in the present analysis for the corresponding IPI skill, Cox found the percent of systematic errors to be 8%.[15] Four of the five systematic errors found in the present study

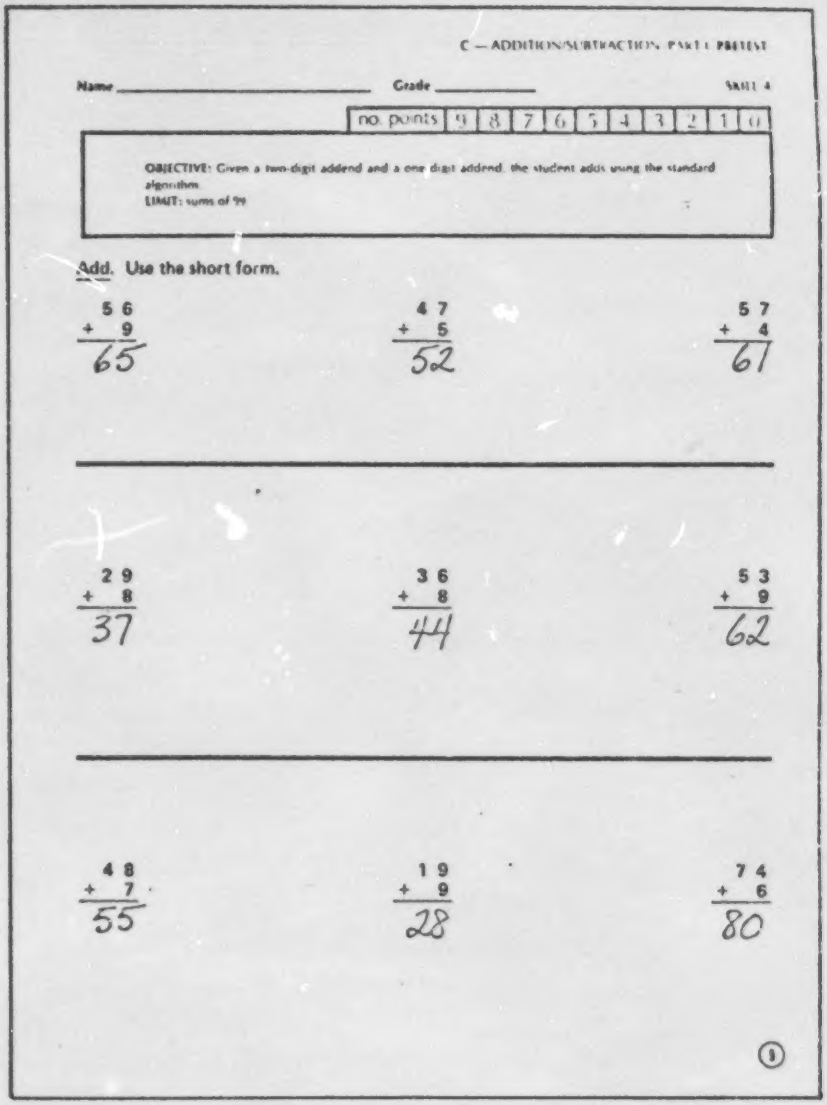
---

[12] Ibid., p. 35.

[13] Ibid., p. 34.

[14] Ibid., p. 31.

[15] Ibid.



C — ADDITION/SUBTRACTION, PART 1 PRETEST

Name _____ Grade _____ SKILL 4

no. points	9	8	7	6	5	4	3	2	1	0

OBJECTIVE: Given a two-digit addend and a one digit addend, the student adds using the standard algorithm.
LIMIT: sums of 99

Add. Use the short form.

56 + 9 = 65	47 + 5 = 52	57 + 4 = 61
29 + 8 = 37	36 + 8 = 44	53 + 9 = 62
48 + 7 = 55	19 + 9 = 28	74 + 6 = 80

①

Figure 2. C-Addition/Subtraction-4 Pretest with Answers

Table 2. C-Addition/Subtraction-4 Pretest Errors

Error				Frequency
Examples			Description	Number (percent)
56 + 9 = 515	47 + 5 = 412	57 + 4 = 511	The sum of the ones' column is placed in the answer without renaming. The ten's digit is placed in the answer to the left of the sum of the ones' column.*	6 (8.7)
56 + 9 = 20	47 + 5 = 16	57 + 4 = 16	The digits in the top addend and in the bottom addend are added separately disregarding place-values. (56 + 9 = 5 + 6 + 9 = 20)*	4 (5.8)
56 + 9 = 55	47 + 5 = 42	57 + 4 = 51	The ten's digit of the sum of the ones' column is not added to the tens' column.*	2 (2.9)
56 + 9 = 15	47 + 5 = 12	57 + 4 = 11	The ones' column is added and the sum placed in the answer. The tens' column is ignored.*	2 (2.9)
56 + 9 = 05	47 + 5 = 02	57 + 4 = 01	The ones' column is added, and the one's digit of that sum is placed in the answer. All ten's digits are ignored and a zero is placed in the ten's column of the answer.	1 (1.4)
			Fact errors	3 (4.3)
			Three or more random errors	2 (2.9)
			Two errors	9 (13.0)
			One or no errors	37 (53.6)
			Did not attempt pretest	3 (4.3)

n = 69

* Also noted by Linda S. Cox, (1974).

are also reported by Cox in her results on the Addition, Level 2 skill.[16] Indeed, the most prevalent systematic error in both studies is the same. Two of the systematic errors Cox reported for Addition, Level 2 were not found in the present study, and one of the systematic errors described in the present study does not appear in Cox's report.

C-Addition/Subtraction-7. Twenty-two of the 70 tests reviewed, or 31.4%, showed systematic errors. Three of the tests included both a systematic error and three or more fact errors. One test included two systematic errors, the first and last systematic error described in Table 3. As was the case in the previous two skills, the number of tests with systematic errors far exceeds the number of tests with three or more fact errors as well as the number of tests with three or more random errors.

Cox considered the addition of two two-digit numbers with renaming from the ones' to the tens' column in her Addition, Level 4 skill. Her measure of post-instructional systematic error rate among the second, third, and fourth grade normal population averages 9% for this skill.[17] For Cox, this is a drop from the Level 2 rate of 11%.[18] The highest rate of systematic error Cox noted for this skill was the 15% rate among second graders,[19] the grade from which most papers for this study came. Only three of the eight systematic errors reported by Cox were noted in the present study. However, the two syctematic errors reported as most frequent

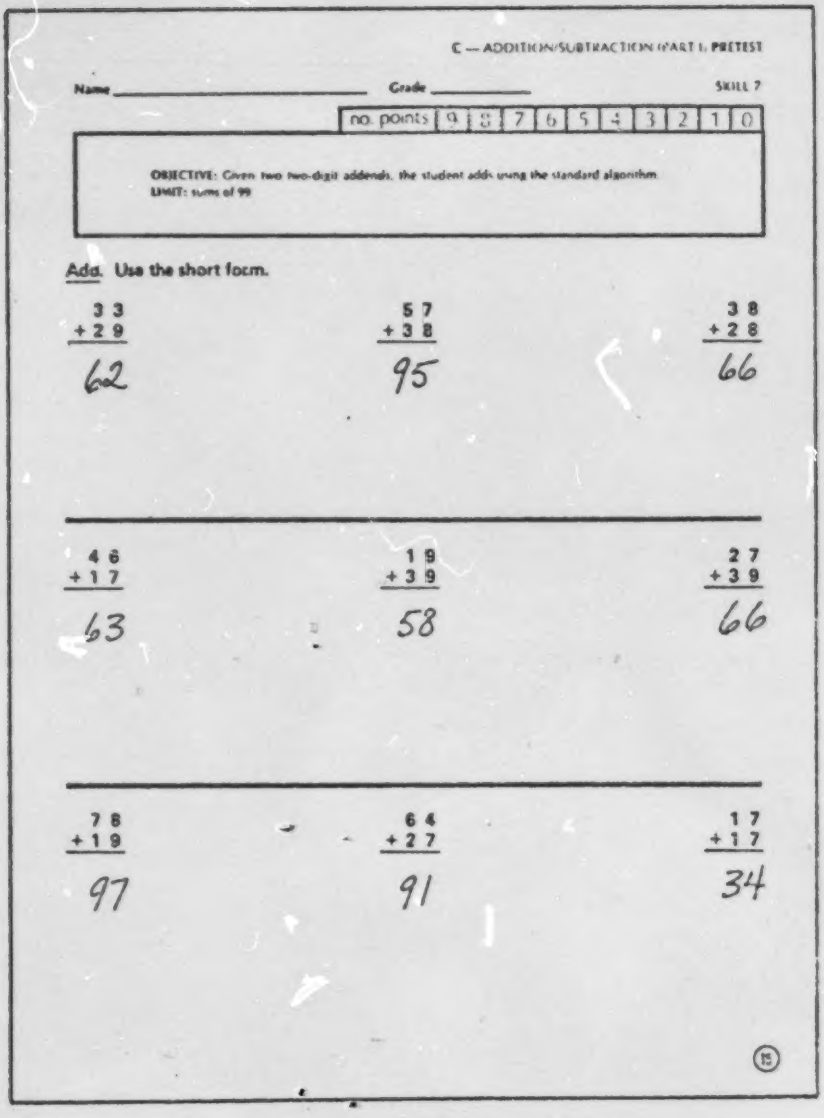
---

[16] Ibid., p. 33.

[17] Ibid., p. 31.

[18] Ibid.

[19] Ibid.



C — ADDITION/SUBTRACTION (PART I: PRETEST

Name _____ Grade _____ SKILL 7

no. points	9	8	7	6	5	4	3	2	1	0

OBJECTIVE: Given two two-digit addends, the student adds using the standard algorithm.
LIMIT: sums of 99

Add. Use the short form.

33 + 29 = 62	57 + 38 = 95	38 + 28 = 66
46 + 17 = 63	19 + 39 = 58	27 + 39 = 66
78 + 19 = 97	64 + 27 = 91	17 + 17 = 34

Figure 3. C-Addition/Subtraction-7 Pretest with Answers

Table 3. C-Addition/Subtraction-7 Pretest Errors

Error Examples			Error Description	Frequency Number (percent)
33 + 29 = 52	57 + 38 = 85	38 + 28 = 56	The ten's digit of the sum of the ones' column is not added to the tens' column. The tens' digits of the two addends are added correctly.*	11 (15.7)
33 + 29 = 512	57 + 38 = 815	38 + 28 = 516	The sum of the ones' column is placed in the answer without renaming. The tens' digits of the two addends are added correctly.*	6 (8.6)
33 + 29 = 17	57 + 38 = 23	38 + 28 = 21	The digits in the top addend and in the bottom addend are added separately disregarding place-values. (32 + 29 = 3 + 3 + 2 + 9 = 17)*	4 (5.7)
33 + 29 = 44	57 + 38 = 68	38 + 28 = 48	The digits in the ones' column and the ten's digit of the bottom addend are added. The one's digit of that sum is placed in the ones' column of the answer. The ten's digit of that same sum is added to the ten's digit of the top addend and the resulting sum is placed in the answer. (57 + 38 = 7 + 8 + 3 + 50 = 68)	1 (1.4)
33 + 29 = 51	57 + 38 = 81	38 + 28 = 51	A "1" is always placed in the ones' column of the answer. The tens' digits of the two addends are added correctly.	1 (1.4)
			Fact errors	6 (8.6)
			Three or more random errors	2 (2.8)
			Two errors	7 (10.0)
			One or no errors	29 (41.4)
			Did not attempt pretest	7 (10.0)

n = 70‡

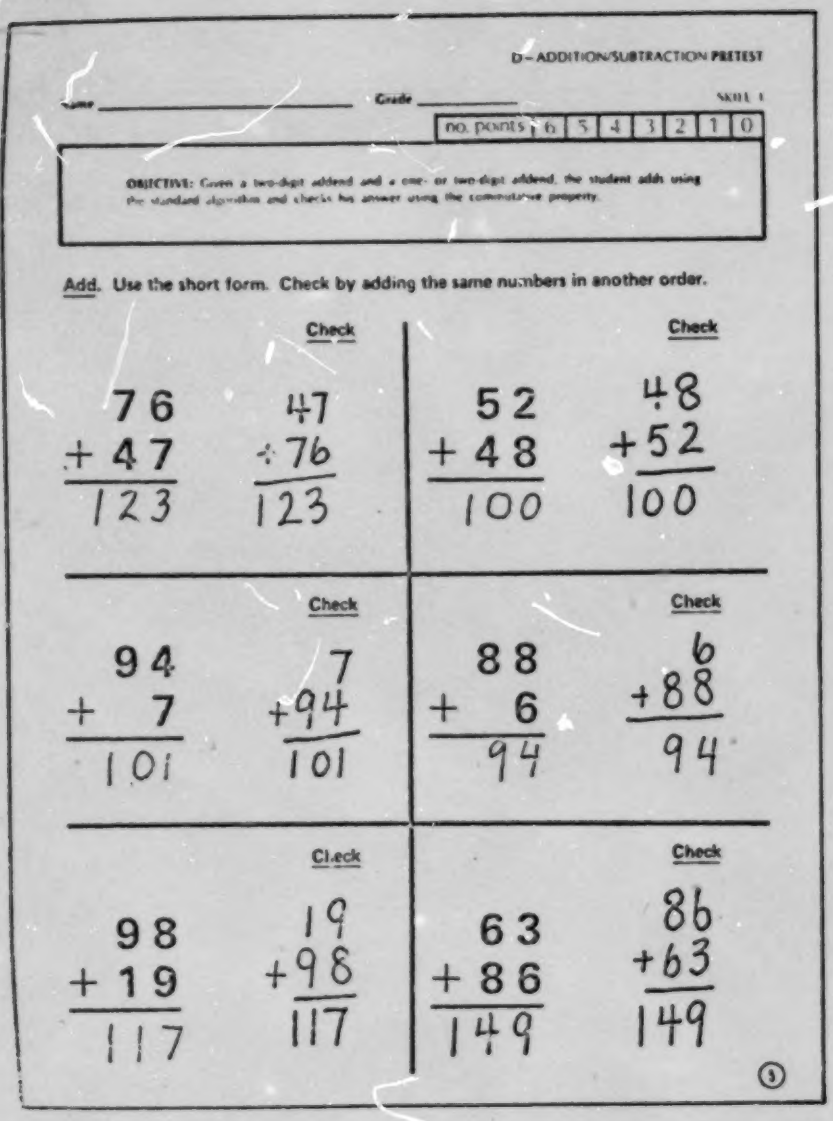
* Also noted by Linda S. Cox, (1974).

‡ The sum of the numbers is greater than 70, and the percents total more than 100, because one or more students were counted in more than one category.

by Cox were also found to be the most frequent in this study. The absence of some of the systematic errors reported by Cox can perhaps be explained by the sequence in the IPI continuum. One of the systematic errors Cox notes is an application of the multiplication algorithm to an addition problem. Since students working on C-Addition/Subtraction-7 have not yet been exposed to the multiplication algorithm, this error would hardly seem likely. Also since the IPI student working on the pretest for C-Addition/Subtraction-7 has not been instructed in renaming from the ones' column, "carrying" a renamed ten to the hundreds' column (an error noted by Cox) is rather unlikely. Two of the systematic errors documented in the present study were not found by Cox.

D-Addition/Subtraction-3. This IPI skill is very similar to the C-Addition/Subtraction-7 skill. There are two differences: (1) the D level skill may require regrouping from the tens' to the hundreds' column, and (2) the D level skill requires the student to check his/her work by using the commutative property. In analyzing the tests, the checking aspect of each item was ignored; only the given addition problems were considered in analyzing the skill. Since the student has received instruction in regrouping from the ones' to the tens' column at C level, the results might be expected to match Cox's results on a similar skill. Unfortunately, Cox does not include a skill exactly comparable to the D-Addition/Subtraction-3 skill. The closest Cox comes to this skill is the same Addition, Level 4 skill used as a comparison for C-Addition/Subtraction-7.

The papers of 100 students were reviewed. Only three students, 3%, made one of the two systematic errors described in Table 4. No test was tallied in more than one category. Even though the number of systematic



D–ADDITION/SUBTRACTION PRETEST

Name _____ Grade _____ SKILL 1

no. points	6	5	4	3	2	1	0

OBJECTIVE: Given a two-digit addend and a one- or two-digit addend, the student adds using the standard algorithm and checks his answer using the commutative property.

Add. Use the short form. Check by adding the same numbers in another order.

	Check		Check
76 + 47 = 123	47 + 76 = 123	52 + 48 = 100	48 + 52 = 100
94 + 7 = 101	7 + 94 = 101	88 + 6 = 94	6 + 88 = 94
98 + 19 = 117	19 + 98 = 117	63 + 86 = 149	86 + 63 = 149

3

Figure 4. D-Addition/Subtraction-3 Pretest with Answers

Table 4. D-Addition/Subtraction-3 Pretest Errors

Error: Examples			Error: Description	Frequency: Number (percent)
6 1 ~~1~~6 + 47 29	88 + 6 82	5 1 ~~6~~3 + 86 37	Subtraction is performed instead of addition. When renaming is required in the ones' column, it is performed. In the tens' column, the smaller digit is subtracted from the larger.*	2 (2.0)
76 + 47 113	88 + 6 84	63 + 86 149	The ten's digit of the sum of the ones' column is not added to the tens' column. Any ten's digits in the addends are added correctly.*	1 (1.0)
			Fact errors	0 (0.0)
			Three or more random errors	0 (0.0)
			Two errors	4 (4.0)
			One or no errors	82 (82.0)
			Did not attempt pretest	11 (11.0)

n = 100

* Also noted by Linda S. Cox, (1974).

errors was small, there were again even fewer papers with three fact errors or with three or more random errors.

The papers analyzed for D-Addition/Subtraction-3 were from the third, fourth, fifth and sixth grades, the largest number coming from fourth graders. The 3% rate of systematic error found in the present study is comparable to the 1% rate of error Cox found among the normal 4th grade population.[20] Both of the errors identified in the present study were also noted by Cox in her report on Addition, Level 4.[21] However, Cox noted eight systematic errors for Level 4, while only two were found for the IPI skill.

D-Addition/Subtraction-4. This IPI skill requires the addition of a three-digit number and a one-, two-, or three-digit number with regrouping from the ones' and tens' columns permitted. Of the 99 papers examined, only two had a systematic error. The two errors, only one of which was also reported by Cox, are described in Table 5. Again, the number of tests with systematic errors, although small, exceeds the number of tests with either three or more fact errors or three or more random errors.

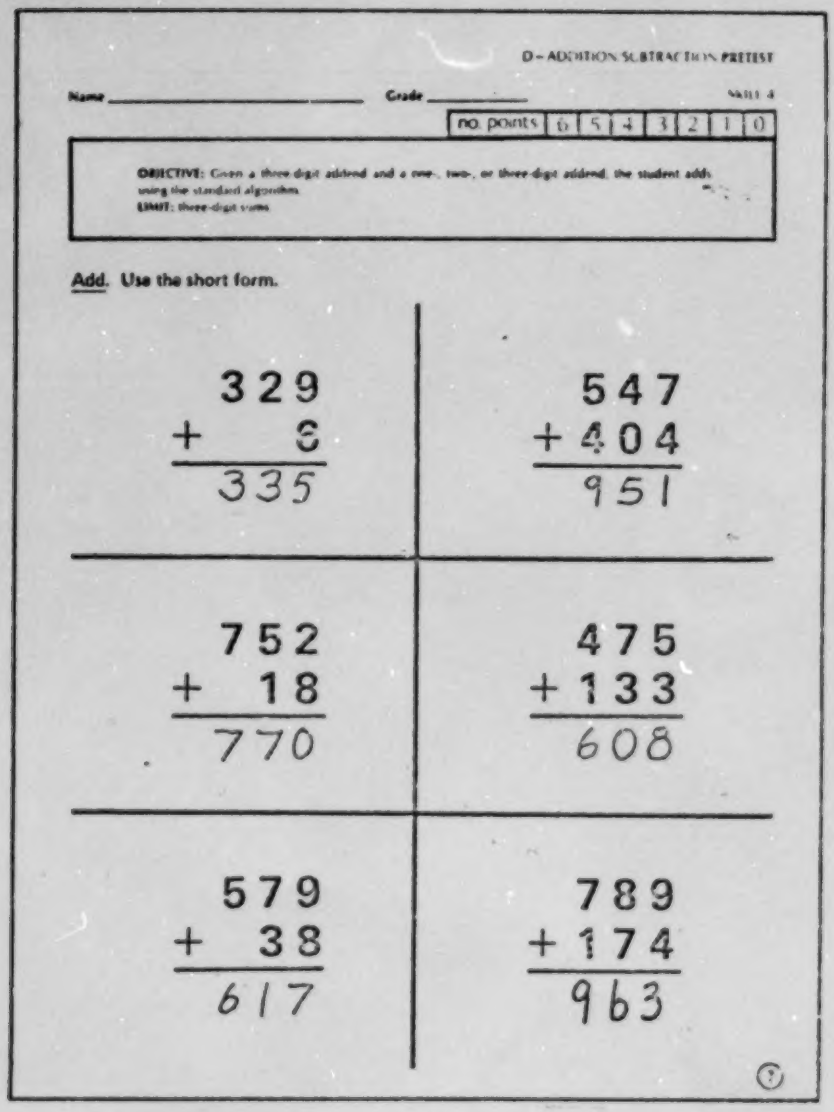
The rate of systematic errors on the pre-instructional IPI pretests seems comparable to Cox's report of the rate of post-instructional systematic errors, between 1 and 2%.[22] The second systematic error listed in Table 5 is similar to, while not precisely the same as, one identified by Cox. The second error seems to be the result of a partial application of a multiplication-like algorithm where only one "partial product" is

---

[20] Ibid.

[21] Ibid., p. 35.

[22] Ibid., p. 31.



D – ADDITION SUBTRACTION PRETEST

Name _____ Grade _____ SKILL 4

no. points	6	5	4	3	2	1	0

OBJECTIVE: Given a three-digit addend and a one-, two-, or three-digit addend, the student adds using the standard algorithm.
LIMIT: three-digit sums

Add. Use the short form.

329 + 6 = 335	547 + 404 = 951
752 + 18 = 770	475 + 133 = 608
579 + 38 = 617	789 + 174 = 963

Figure 5. D-Addition/Subtraction-4 Pretest with Answers

Table 5. D-Addition/Subtraction-4 Pretest Errors

Error				Frequency
Examples			Description	Number (percent)
326 + 6 = 322	547 + 404 = 941	789 + 174 = 853	The ten's digit of the sum of the ones' column is not added to the tens' column. Similarly, the hundred's digit of the sum of the tens' column is not added to the hundreds' column.*	1 (1.0)
326 + 6 = 992	547 + 404 = 991	789 + 174 = 1233	All but the one's digit of the bottom addend are ignored. The one's digit of the bottom addend is added to each digit of the top addend. Renaming, when necessary, is performed as usual.	1 (1.0)
			Fact errors	0 (0.0)
			Three or more random errors	0 (0.0)
			Two errors	4 (4.0)
			One or no errors	91 (91.8)
			Did not attempt pretest	2 (2.0)
				n = 99

* Also noted by Linda S. Cox, (1974).

recorded. It was noted above that this type of error was reported by Cox for a skill comparable to C-Addition/Subtraction-4. The point was made that IPI students had not yet been exposed to the multiplication algorithm. However, the multiplication algorithm is treated in the C-Multiplication unit, so that a student reaching the D-Addition/Subtraction pretest would have performed the multiplication algorithm with a one-digit multiplier. Thus, it is reasonable for this type of systematic error to begin appearing on IPI students' tests.

Discussion of Findings for Addition Skills. The types of systematic errors that were identified ranged from quite obvious errors to some that were more difficult to detect. While some systematic errors may have gone undetected and may have been counted among the random errors, all pretests containing three or more random errors were subject to review by two, and more often three, individuals. Since the pretest booklet containing the first three of the five addition skills also contained a test of the basic addition combinations, the reviewer could identify each paper as having been completed by a student who knew his facts well, had some difficulty with basic facts, or had a great deal of difficulty with basic facts. The papers of students who did well on basic facts but appeared to have random errors on later skills were most carefully scrutinized.

Some systematic errors showed misconceptions about the nature of addition, the function of place value, and/or the process of renaming. It is important to note that in the vast majority of cases, addition facts were correct but the process of applying the addition algorithm was faulty. Drill in basic facts, while necessary, is not the panacea some teachers seem to wish it to be.

Few students' papers were tallied in two categories. This would indicate that students generally adopted a technique and held to that process. This is also evidenced by the fact that it was not unusual for a student who evidenced one systematic error on a skill to apply that same pattern to items on similar skills encountered later in the pretest booklet.

It was of interest to note that the number of distinct systematic errors as well as the percent of papers containing systematic errors dropped sharply from C level skills to D level skills. Indeed, the percent of systematic errors on the two D level pretests is surprisingly close to the rate of post-instructional systematic errors noted by Cox. Once students had been exposed to instruction in the additional algorithm at C level, they apparently made fewer systematic errors when applying that algorithm to problems with more digits in each addend. Cox points out that among the addition skills the percent of systematic errors declined as grade level increased.[23] Cox suggests that this may be due to increased maturity of thought.[24] However, since age and amount of exposure to instruction and opportunity to practice are apt to be closely related, it is difficult to suggest which factor, if either, is most important. It would be of interest to compare the rates of systematic error made on the same IPI pretest (same amount of curricular exposure) by students of various grade levels. Data from the present study could be analyzed in that manner; however it might be desirable to increase the number of tests from various grade levels.

---

[23] Ibid., p 49.

[24] Ibid.

Subtraction Skills

Pretests for five subtraction skills were analyzed. The skills ranged from subtraction of a one or two-digit number from a two-digit number with no regrouping to subtraction of a one-, two-, three-, or four-digit number from a four-digit number with regrouping from any but the thousands' column. Figures 6-10, which follow, show the pretests with the correct answer for each item. Tables 6-10, below, include descriptions of each of the systematic errors noted and the number and percent of tests falling in each of the six previously defined categories. A total of 443 tests were analyzed: 81 for C-Addition/Subtraction-8, 81 for C-Addition/Subtraction-10, 82 for C-Addition/Subtraction-11, 98 for D-Addition/Subtraction-7, and 101 for D-Addition/Subtraction-9.

C-Addition/Subtraction-8. Of the 81 tests analyzed, 8.6% contained one of the three distinct systematic errors listed in Table 6. For this skill, described in Figure 6, no test was tallied in more than one category.

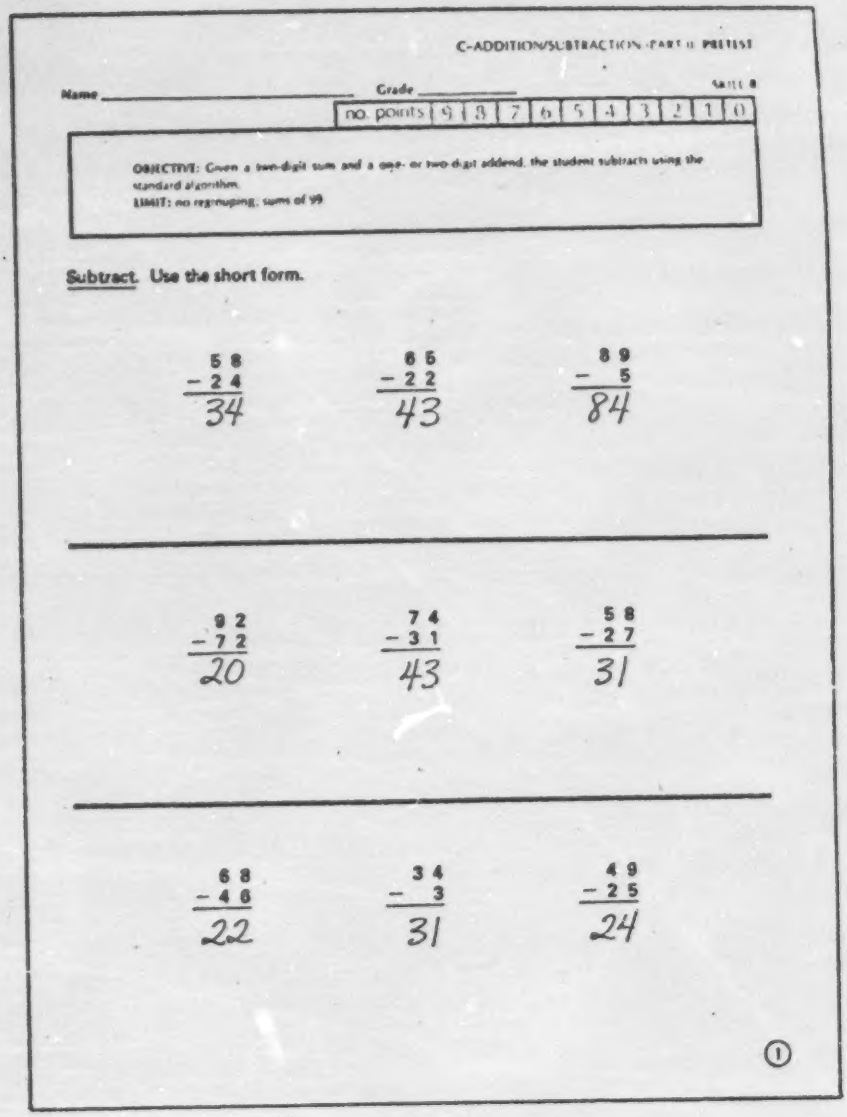
Cox, in her post-instructional survey, found a systematic error rate of less than 2%.[25] The error Cox listed as most frequent is also the one Table 6 lists as the most frequent. The two other systematic errors described in Table 6 differ from the two described by Cox for her related skill, Subtraction, Level 3.[26]

C-Addition/Subtraction-10. On the C-Addition/Subtraction-10 skill that involves regrouping for subtraction more systematic errors were

---

[25] Ibid., p. 55.

[26] Ibid., p. 60.



C-ADDITION/SUBTRACTION-PART II PRETEST

Name _____ Grade _____ Skill 8

no. points	9	8	7	6	5	4	3	2	1	0

OBJECTIVE: Given a two-digit sum and a one- or two-digit addend, the student subtracts using the standard algorithm.
LIMIT: no regrouping; sums of 99

Subtract. Use the short form.

58 − 24 = 34	66 − 22 = 43	89 − 5 = 84
92 − 72 = 20	74 − 31 = 43	58 − 27 = 31
68 − 46 = 22	34 − 3 = 31	49 − 25 = 24

①

Figure 6. C-Addition/Subtraction-8 Pretest with Answers

Table 6. C-Addition/Subtraction-8 Pretest Errors

Error: Examples			Error: Description	Frequency: Number (percent)
58 - 24 82	65 - 22 87	89 - 5 94	Addition, rather than subtraction, is performed.*	5 (6.2)
4 1 7 58 - 24 24	1 1 8 - 16 2	7 1 3 - 32 41	In the ones' column, subtraction is performed correctly. The ten's digit of the sum is unnecessarily renamed to the next lower digit.	1 (1.2)
58 - 24 24	28 - 16 12	83 - 32 31	In the ones' column, subtraction is performed correctly. The tens' digit of the known addend is placed in the tens' column of the answer.	1 (1.2)
			Fact errors	1 (1.2)
			Three or more random errors	4 (4.9)
			Two errors	5 (6.2)
			One or no errors	62 (76.5)
			Did not attempt pretest	2 (2.5)

n = 81

* Also noted by Linda S. Cox, (1974).

documented (see Table 7) and a greater percent of the 81 papers, 56.6%, evidenced systematic errors than were documented for either the first subtraction skill, or the first addition skill requiring regrouping. Table 7 indicates that the most frequent error, that of subtracting the lesser digit from the greater digit in each column, accounted for 72% of all the systematic errors made on the pretest. Although ten other systematic errors were described, they were each employed by only one or two students. One test was tallied both in the fourth systematic error category and in the three or more basic facts category. Again, the number of tests with systematic errors was far greater than the number of tests with three or more random errors or with three or more fact errors.

Cox in her report on data on a corresponding skill, Subtraction, Level 2, noted that 13% of the papers she analyzed contained systematic errors.[27] For grade three, the grade from which most of the IPI pretests for his skill came, Cox found systematic errors on 23% of the papers.[28] The vast difference in the percent of systematic errors found in the two studies is probably due to the pre-instructional post-instructional distinction made previously. It is interesting to note that the most frequent systematic error reported in Table 7 is also the most frequent systematic error reported by Cox. In the present study, this error accounted for 72% of all the systematic errors documented. In Cox's study, the error accounted for 63% of all systematic errors documented.[29] While Cox described 14 distinct systematic errors for this skill, only 11 distinct systematic errors are reported in Table 7.

---

[27] Ibid., p. 55

[28] Ibid.

[29] Ibid., pp. 58-59.

C-ADDITION/SUBTRACTION (PART II) PRETEST

Name _____ Grade _____ SKILL 10

no. points	9	8	7	6	5	4	3	2	1	0

OBJECTIVE: Given a two-digit sum and any one-digit addend, the student subtracts using the standard algorithm.

Subtract. Use the short form.

51 − 6 = 45	43 − 8 = 35	76 − 9 = 67
22 − 4 = 18	31 − 5 = 26	80 − 7 = 73
95 − 7 = 88	57 − 8 = 49	46 − 7 = 39

Figure 7. C-Addition/Subtraction-10 Pretest with Answers

Table 7. C-Addition/Subtraction-10 Pretest Errors

Error				Frequency
Examples			Description	Number (percent)
51 − 6 = 55	43 − 8 = 45	76 − 9 = 73	No renaming is performed. The lesser one's digit of the sum is subtracted from the greater one's digit of the known addend. The ten's digit of the sum is placed in the tens' column of the answer.*	33 (40.7)
51 − 6 = 05	43 − 8 = 05	76 − 9 = 03	No renaming is performed. The lesser one's digit of the sum is subtracted from the greater one's digit of the known addend. A zero is recorded in the tens' column of the answer.*	2 (2.5)
51 − 6 = 50	43 − 8 = 40	76 − 9 = 70	A zero is placed in the ones' column of the answer. The ten's digit of the sum is placed in the tens' column of the answer.*	2 (2.5)
4 81 − 6 = 45	3 43 − 8 = 35	6 76 − 9 = 63	The one's digit of the sum is renamed correctly. However the lesser one's digit of the sum is then subtracted from the greater one's digit of the known addend. The renamed ten's digit of the sum is then placed in the tens' column of the answer.*	2 (2.5)
4 81 − 6 = 5	3 43 − 8 = 5	6 76 − 9 = 7	The one's digit of the sum is correctly renamed and subtraction is performed in the ones' column. No digit is placed in the tens' column of the answer.*	1 (1.2)
51 − 6 = 57	43 − 8 = 51	76 − 9 = 85	Addition, instead of subtraction, is performed.*	1 (1.2)
51 − 6 = 6	43 − 8 = 43	76 − 9 = 9	No subtraction is performed. Either the sum or known addend is placed in the answer.*	1 (1.2)

* Also noted by Linda S. Cox, (1974).

Table 7. C-Addition/Subtraction-10 Pretest Errors (con't)

Error				Frequency
Examples			Description	Number (percent)
$\begin{array}{r} 4\not{5}1^{\circ} \\ -\ 6 \\ \hline 44 \end{array}$	$\begin{array}{r} 3\not{4}3^{\circ} \\ -\ 8 \\ \hline 322 \end{array}$	$\begin{array}{r} 6\not{7}6^{\circ} \\ -\ 9 \\ \hline 651 \end{array}$	The one's digit of the sum is renamed by placing a zero to its right. The ten's digit of the sum is reduced in value by one. The one's digit of the known addend is subtracted from the multiple of ten that was created in the ones' column and the difference is recorded in the answer. The renamed ten's digit of the sum is placed in the answer to the left of the previously recorded digit(s).	1 (1.2)
$\begin{array}{r} 4\not{5}1' \\ -\ 6 \\ \hline 45 \end{array}$	$\begin{array}{r} 3\not{4}3'' \\ -\ 8 \\ \hline 33 \end{array}$	$\begin{array}{r} 6\not{7}6'' \\ -\ 9 \\ \hline 62 \end{array}$	The one's digit of the sum is renamed to eleven. The ten's digit of the sum is reduced in value by one. Subtraction is then performed correctly in both columns.	1 (1.2)
$\begin{array}{r} 51 \\ -\ 6 \\ \hline 44 \end{array}$	$\begin{array}{r} 43 \\ -\ 8 \\ \hline 32 \end{array}$	$\begin{array}{r} 76 \\ -\ 9 \\ \hline 61 \end{array}$	The digit placed in the ones' column of the answer is the difference between ten and the one's digit of the sum. The digit placed in the tens' column of the answer is one less than the ten's digit of the sum.	1 (1.2)
$\begin{array}{r} 51 \\ -\ 6 \\ \hline 47 \end{array}$	$\begin{array}{r} 43 \\ -\ 8 \\ \hline 31 \end{array}$	$\begin{array}{r} 76 \\ -\ 9 \\ \hline 65 \end{array}$	Either addition or subtraction is performed in the ones' column. If addition is performed, there is no renaming of this sum; the ones' digit only is recorded. The digit placed in the tens' column of the answer is one less than the ten's digit of the sum	1 (1.2)

Table 7. C-Addition/Subtraction-10 Pretest Errors (con't)

Error		Frequency
Examples	Description	Number (percent)
	Fact errors	3 (2.6)
	Three or more random errors	4 (4.9)
	Two errors	5 (6.1)
	One or no errors	17 (20.9)
	Did not attempt pretest	7 (8.6)

n = 81[1]

[1] The sum of the numbers is greater than 81, and the percents total more than 100, because one or more students were counted in more than one category.

C-Addition/Subtraction-11. A reader who feels that the last skill evoked a great number of systematic errors will find that Table 8 describes twelve distinct systematic errors found on 50 of the 82 pretests that were reviewed. Again, one of the systematic errors accounts for the vast majority, 70%, of all the systematic errors described in Table 8. Perhaps not surprising, is the fact that the error is exactly the same as the most frequent error reported in Table 7; i.e., the lesser digit in each column is subtracted from the greater digit. None of the other systematic errors occurs with anywhere near this frequency. The majority of the 12 systematic errors involve faulty "renaming." One of the tests contained both a systematic error and three or more fact errors. Another pretest contained two systematic errors and three or more basic fact errors.

The Cox skill which is comparable to this IPI skill is Subtraction, Level 4. Cox finds the same systematic error reported as most frequent in this study as the most frequent in her own study. In Cox's study this most frequent systematic error accounted for almost 83% of all systematic errors reported on the skill.[30] Even in her post-instruction survey, Cox found that the average percent of systematic errors over the second, third and fourth grade normal populations was 21%.[31] For the Subtraction, Level 4 skill Cox documented only 6 distinct systematic errors, while 12 are described in Table 8.

---

[30] Ibid., p. 61

[31] Ibid., p. 55.

C-ADDITION/SUBTRACTION (PART II) PRETEST

Name _____ Grade _____ SKILL 11

no. points	9	8	7	6	5	4	3	2	1	0

OBJECTIVE: Given a two-digit sum and a two-digit addend, the student subtracts using the standard algorithm.

Subtract. Use the short form.

84 − 28 = 56	54 − 27 = 27	75 − 46 = 29
26 − 18 = 8	63 − 26 = 37	90 − 85 = 5
51 − 25 = 26	42 − 19 = 23	37 − 29 = 8

①

Figure 8. C-Addition/Subtraction-11 Pretest with Answers

Table 8. C-Addition/Subtraction-11 Pretest Errors

Error				Frequency
Examples			Description	Number (percent)
84 − 28 = 64	54 − 27 = 33	75 − 46 = 31	No renaming is performed. The lesser one's digit of the sum is subtracted from the greater one's digit of the known addend. The tens' digits are subtracted correctly.*	32 (39.0)
84 − 28 = 60	54 − 27 = 30	75 − 46 = 30	A zero is placed in the ones' column of the answer. Subtraction is performed in the tens' column without renaming the sum.*	4 (4.9)
84 − 28 = 66	54 − 27 = 37	75 − 46 = 39	The one's digit of the sum is renamed and subtraction in the ones' column is performed correctly. However subtraction in the tens' column is performed as if the ten's digit of the sum had not been renamed.*	3 (3.7)
84 − 28 = 76	54 − 27 = 47	75 − 46 = 69	The one's digit of the sum is renamed and subtraction in the ones' column is performed correctly. The renamed ten's digit of the sum is placed in the tens' column of the answer.*	1 (1.2)
84 − 28 = 64	54 − 27 = 34	75 − 46 = 35	The one's digit of the sum is placed in the ones' column of the answer. Subtraction is performed in the tens' column without renaming.*	1 (1.2)
7 84 − 28 = 54	4 54 − 27 = 23	6 75 − 46 = 21	The lesser one's digit of the sum is subtracted from the greater one's digit of the known addend. The ten's digit of the sum is reduced in value by one and subtraction is performed in the tens' column.	1 (1.2)

* Also noted by Linda S. Cox, (1974).

Table 8. C-Addition/Subtraction-11 Pretest Errors (con't)

Error Examples			Error Description	Frequency Number (percent)
84 - 28 112	54 - 27 81	75 - 46 121	Addition, instead of subtraction, is performed.	1 (1.2)
184 + 28 96	454 - 27 67	675 - 46 109	The renaming of the sum is carried out correctly. Subtraction is performed correctly in the ones' column but addition is performed in the tens' column.	1 (1.2)
64 - 28 46	34 - 27 17	35 - 46 19	The one's digit of the sum is increased by ten. Subtraction in the ones' column is performed correctly. The ten's digit of the sum is renamed by subtracting from it the ten's digit of the known addend. The absolute value of the difference between the incorrectly renamed ten's digit of the sum and the ten's digit of the known addend is placed in the tens' column of the answer.	1 (1.2)
1 11 84 - 28 73	4 11 54 - 27 44	6 11 15 - 46 65	The one's digit of the sum is renamed to eleven and subtraction is performed in the ones' column. The ten's digit of the sum is reduced in value by one and is written in the tens' column of the answer.	1 (1.2)
84 - 28 62	54 - 27 33	75 - 46 34	The digit placed in the ones' column of the answer is the difference between ten and the one's digit of the known addend. Subtraction is performed in the tens' column without renaming the sum.	1 (1.2)
84 - 28 62	54 - 27 31	75 - 46 31	Addition is performed in the ones' column. If the sum is greater than 10, only the one's digit is recorded. Subtraction is performed in the tens' column without renaming.	1 (1.2)

Table 8. C-Addition/Subtraction-11 Pretest Errors (con't)

Error		Frequency
Examples	Description	Number (percent)
	Fact errors	3 (3.7)
	Three or more random errors	4 (4.9)
	Two errors	2 (2.4)
	One or no errors	15 (18.3)
	Did not attempt pretest	13 (15.8)

n = 82‡

‡ The sum of the numbers is greater than 82, and the percents total more than 100, because one or more students were counted in more than one category.

D-Addition/Subtraction-7. Among the 98 tests analyzed, 28 included one systematic error and 2 included two systematic errors. The eight distinct systematic errors, described in Table 9, were evidenced on approximately 31% of the papers. Again, the most frequent systematic error found was the error that was also found as the most frequent on the C-Addition/Subtraction-10 and 11 pretests. Although the percent of tests with only two errors or with three or more random errors is somewhat greater than for the skills reported on above, the percent of tests with systematic errors is still greater than the percent in those two categories.

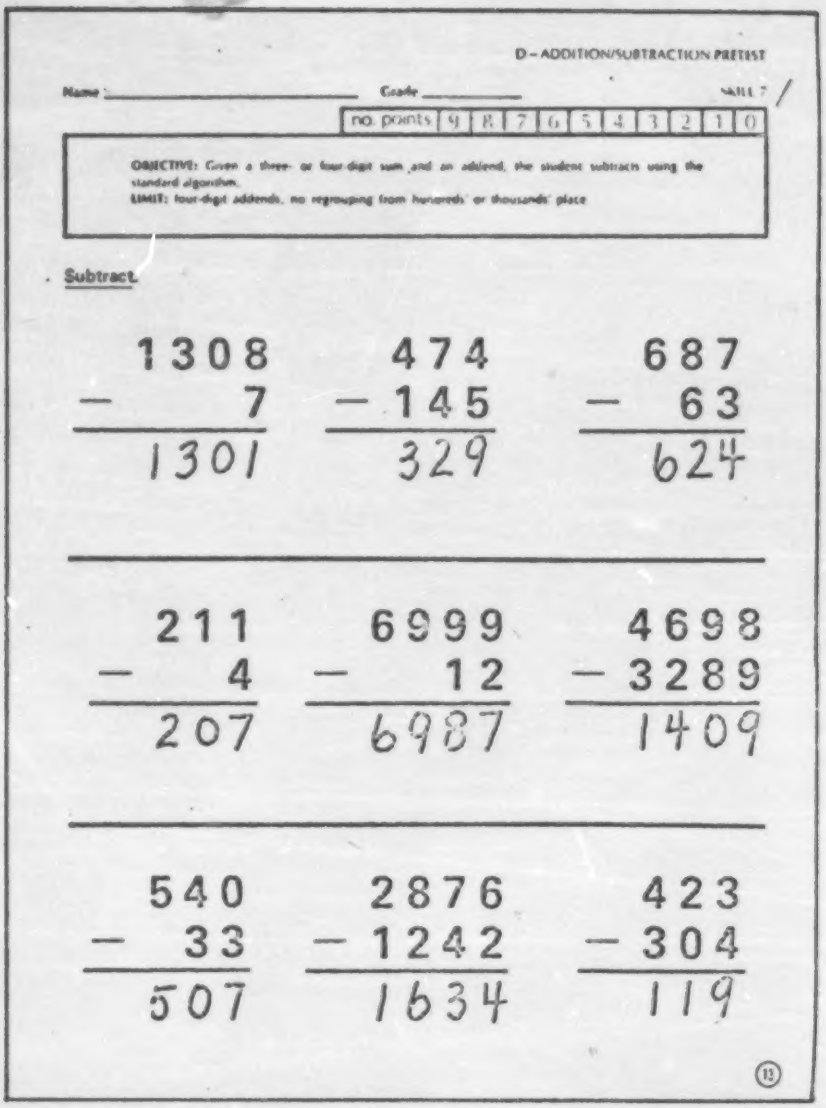
D-Addition/Subtraction-7 does not correspond exactly with any of the Levels used by Cox. Cox's Subtraction, Level 5 skill is limited to subtraction of a two-digit number from a three-digit number with regrouping only to the ones' column.[32] This D level IPI skill has the same regrouping limit, but the known addend may have three or even four digits. Since it is the regrouping process that causes the most difficulty, Cox's Subtraction, Level 5 is probably a fair choice for comparison. Six of the eight systematic errors described in Table 9 are among the eleven systematic errors Cox listed for her Level 5 skill.[33] The systematic error we found to be most frequent was the same as the one Cox found to be most prevalant. The average percent of systematic errors Cox found on tests for this level from third, fourth, and fifth graders was 8%.[34]

---

[32] Ibid., p. 53.

[33] Ibid., pp. 62-63.

[34] Ibid., p. 55.



D – ADDITION/SUBTRACTION PRETEST

Name: _____ Code _____ SKILL 7 /

no. points	9	8	7	6	5	4	3	2	1	0

OBJECTIVE: Given a three- or four-digit sum and an addend, the student subtracts using the standard algorithm.
LIMIT: four-digit addends, no regrouping from hundreds' or thousands' place

Subtract.

1308 − 7 = 1301	474 − 145 = 329	687 − 63 = 624
211 − 4 = 207	6999 − 12 = 6987	4698 − 3289 = 1409
540 − 33 = 507	2876 − 1242 = 1634	423 − 304 = 119

(13)

Figure 9. D-Addition/Subtraction-7 Pretest with Answers

Table 9. D-Addition/Subtraction-7 Pretest Errors

Error: Examples			Error: Description	Frequency: Number (percent)
423 − 304 = 121	474 − 145 = 331	211 − 4 = 213	No renaming is performed. The lesser of the ones' digits is subtracted from the greater and the difference is placed in the ones' column of the answer. The remaining columns are subtracted without renaming.*	10 (10.2)
423 − 304 = 129	474 − 145 = 339	211 − 4 = 217	The one's digit of the sum is renamed when necessary and subtraction is performed correctly in the ones' column. However, subtraction in the tens' column is performed as if the ten's digit of the sum had not been renamed.*	6 (6.1)
423 − 304 = 19	474 − 145 = 229	211 − 4 = 107	If renaming from ten's to one's is necessary, each digit of the sum to the left of the ones' place is reduced in value by one. Correct subtractions are performed in each column.	4 (4.1)
1308 − 7 = 0001	687 − 63 = 024	540 − 33 = 007	Subtraction is performed only in columns where there are digits in the known addend. In the remaining columns, a zero is placed in the answer.*	4 (4.1)
423 − 304 = 727	474 − 145 = 619	211 − 4 = 215	Addition, instead of subtraction, is performed.*	3 (3.1)
474 − 145 = 239	211 − 4 = 117	4698 − 3289 = 419	In renaming the one's digit of the sum, the left-most digit of the sum (rather than the ten's digit) is reduced in value by one. Correct subtractions are performed in each column.*	3 (3.1)

* Also noted by Linda S. Cox, (1974).

Table 9. D-Addition/Subtraction-7 Pretest Errors (con't)

Error				Frequency
Examples			Description	Number (percent)
474 - 145 330	211 - 4 210	540 - 33 510	A zero is placed in the ones' column of the answer. Subtractions are performed in the remaining columns without renaming.*	1 (1.0)
474 - 145 331	21X - 4 214	5X0 - 33 533	No renaming is performed. In each column either the digit of lesser value is subtracted from the digit of greater value or the digit in the sum is crossed out and the digit in the known addend is placed in the answer.	1 (1.0)
			Fact errors	1 (0.0)
			Three or more random errors	9 (9.2)
			Two errors	6 (6.1)
			One or no errors	47 (47.9)
			Did not attempt pretest	5 (5.1)

n = 98‡

* Also noted by Linda S. Cox, (1974).

‡ The sum of the numbers is greater than 98, and the percents total more than 100, because one or more students were counted in more than one category.

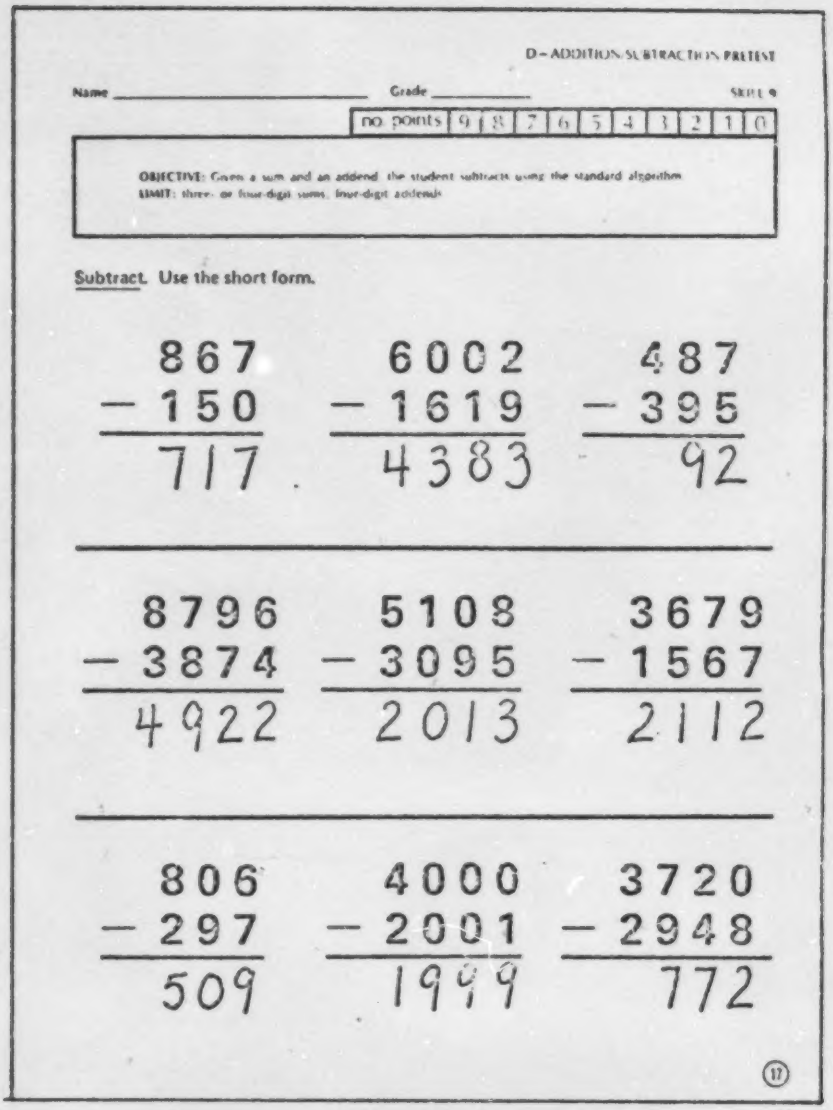
D-Addition/Subtraction-9. As shown in Table 10, there were 55 cases of systematic errors on the 101 pretests that were analyzed. Since eight pretests included two of the eight distinct systematic errors, the percent of pretests with systematic errors was approximately 45%. The fact that this is a higher rate of systematic errors than on the previous skill is likely due to the presence of several items with the digit zero in the tens' and hundreds' column of the sum. Actually, 33% of all the systematic errors tallied for this skill had to do with attempted but faulty renaming of zeros in the sum. It was also noted that five of the eight tests with two systematic errors included one systematic error that had to do with renaming over zeros. Also, on many of the tests tallied in the two errors category or the three or more random errors category the two items, 6002-1619 and 4000-2001, were frequently completed incorrectly. Had there been more items on which the errors could have been repeated, it is probable that there would have been more systematic errors.

D-Addition/Subtraction-9 is comparable to Cox's Subtraction, Level 6. Only four of the eight systematic errors listed in Table 10 can be found among the twelve systematic errors Cox listed for Level 6.[35] However, the systematic errors which Cox found to be the third and second most frequent were, respectively, the first and second most frequent systematic errors found in the present study. Cox listed "multiple errors in regrouping" as the most prevalant systematic error.[36] Since no category of this breadth was included in the present study, no direct comparison can be made.

---

[35] Ibid., pp. 64-65.

[36] Ibid., p. 64.



D–ADDITION/SUBTRACTION PRETEST

Name _____ Grade _____ SKILL 9

no. points	9	8	7	6	5	4	3	2	1	0

OBJECTIVE: Given a sum and an addend, the student subtracts using the standard algorithm.
LIMIT: three- or four-digit sums, four-digit addends

Subtract. Use the short form.

867 − 150 = 717	6002 − 1619 = 4383	487 − 395 = 92
8796 − 3874 = 4922	5108 − 3095 = 2013	3679 − 1567 = 2112
806 − 297 = 509	4000 − 2001 = 1999	3720 − 2948 = 772

17

Figure 10. D-Addition/Subtraction-9 Pretest with Answers

Table 10. D-Addition/Subtraction-9 Pretest Errors

Error				Frequency
Examples			Description	Number (percent)
4^{1}87 - 395 192	8^{1}06 - 297 619	4^{1}0^{1}0 - 2001 2009	When necessary, digits in the sum are renamed to permit subtraction. However when subtractions are performed in the columns to the left, they are performed as if the digits of the sum had not been renamed.*	19 (18.8)
487 - 395 112	806 - 297 691	4000 - 2001 2001	No renaming is performed. In each column the digit of lesser value is subtracted from the digit of greater value.*	13 (12.8)
60^{1}02 - 1619 5003	80^{1}6 - 297 609	4000 - 2001 2000	Whenever a zero appears in the sum, no regrouping is performed and a zero is placed in that column of the answer. When necessary, non-zero digits are renamed to permit subtraction. However when subtractions are performed in the columns to the left, they are performed as if the digit of the sum had not been renamed.	8 (7.9)
60^{1}02 - 1619 5613	80^{1}6 - 297 699	4000 - 2001 2001	In all columns where a zero appears in the sum, or occurs in the sum after renaming, the digit in the known addend is placed in that column of the answer. When necessary, non-zero digits are renamed to permit subtraction. However when zero's intervenes in the sum, subtractions in columns to the left are performed as if the sum had not been renamed.*	8 (7.9)
487 - 395 102	806 - 297 600	8796 - 3874 5022	No renaming is performed. In any column where renaming is required, a zero is placed in the answer.	3 (2.9)

* Also noted by Linda S. Cox, (1974).

Table 10. D-Addition/Subtraction-9 Pretest Errors (con't)

Error: Examples			Error: Description	Frequency: Number (percent)
3 4000 − 2001 = 1009	3 4 5 6002 − 1619 = 2493	6 7 806 − 297 = 419	When a zero or zeros in the sum necessitate renaming, the left-most, non-zero digit of the sum is renamed, repeatedly where necessary.	2 (2.0)
806 − 297 = 691	8196 − 3874 = 5822	3720 − 2948 = 1228	No renaming is performed. In each column either the digit of lesser value is subtracted from the digit of greater value or the digit in the sum is crossed out and the digit in the known addend is placed in the answer.	1 (1.0)
806 − 297 = 1103	4000 − 2001 = 6001	487 − 395 = 882	Addition, instead of subtraction, is performed.*	1 (1.0)
			Fact errors	0 (0.0)
			Three or more random errors	19 (18.8)
			Two errors	4 (4.0)
			One or no errors	19 (18.8)
			Did not attempt pretest	12 (11.9)

n = 101‡

* Also noted by Linda S. Cox, (1974).

‡ The sum of the numbers is greater than 101, and the percents total more than 100, because one or more students were counted in more than one category.

Had we chosen to include such a category, many of the tests tallied in the three or more random errors category would undoubtedly have fallen into it. At the third grade level, Cox found that 64% of the tests she analyzed contained systematic errors.[37] At the fourth grade level, Cox found a 24% rate, at the fifth grade level 5% and at the sixth grade 6%.[38] It is likely that Cox's inclusion of the previously mentioned "multiple errors in regrouping" category tended to produce the high 64% rate for third grade. Our own sample for this skill was composed mainly of fourth graders.[39] If we added to the 45% systematic errors the 18.8% in the three or more random errors category, a 64% "systematic" error rate would result. The percents Cox reports are, therefore, difficult to use for direct comparisons.

Discussion of Findings for Subtraction Skills. As for addition, the types of systematic subtraction errors that were identified ranged from simple to complex. Some papers assigned to the random error category may have contained systematic errors, but two or three individuals reviewed each paper assigned to the random error category.

The most prevalent systematic error was the practice of subtracting a smaller digit in the sum from a larger digit in the addend. Many of the remaining systematic errors involved attempted but faulty renaming. In comparison with the number of pretests containing systematic errors, the number of pretests with three or more items incorrect due to basic

---

[37] Ibid., p. 55.

[38] Ibid.

[39] See Appendix B for the exact distribution.

fact errors was very small. Pretests with systematic errors usually contained just one such error pattern. Where two systematic error patterns were noted, one was almost always associated with a particular type of item (e.g., zeros in the tens' and hundred' place of the sum).

Although there was a noticeable drop in the percent of pretests with systematic errors from C-Addition/Subtraction-10 and 11 to D-Addition/Subtraction-7, the drop was not as great as the drop reported for the same step in the addition skills sequence. Also, the lower rate was not maintained for D-Addition/Subtraction-11. While the D-Level addition skill systematic error rate on the pretests analyzed in the present study was close to Cox's post-instructional systematic error rate, this is not the case for the D-level subtraction skills. The subtraction results are logically not, however, surprising. In subtraction examples with digits of zero in the sum, the renaming process becomes much more complex than any renaming that occurs in a two addend addition example. Thus, while mastery of two two-digit addition problems involves almost all of the skills necessary for two three- or four-digit addition problems, the mastery of subtraction problems involving two digit sums does not involve some skills necessary for subtraction from a three- or four-digit sum.

Generally, the trends in rate of systematic error and the type of systematic error found to be most prevalant in the present study are compatable with those reported by Cox.[40] Although Cox's post instructional

---

[40]Cox, op. cit., pp. 44-51.

study found faulty renaming, not lack of renaming, to be the most prevalant type of systematic error, this is probably due to the difference in timing (pre-instruction versus post-instruction) of the student. work samples. Cox reports that while her average percent of systematic error for her eight addition skills was 5%, the average percent of systematic errors over her six subtraction skills was 13%.[41] For the five addition skills analyzed in the present study the average percent of pretests with systematic errors was approximately 12%; for the five subtraction skills the average percent of pretests with systematic errors was approximately 40%. Thus while the post-instructional percent of systematic errors for subtraction was about double the post-instructional percent of systematic errors for addition, the pre-instructional percent of systematic errors for subtraction was three times the pre-instructional percent for addition.

---

[41] Ibid., p. 48.

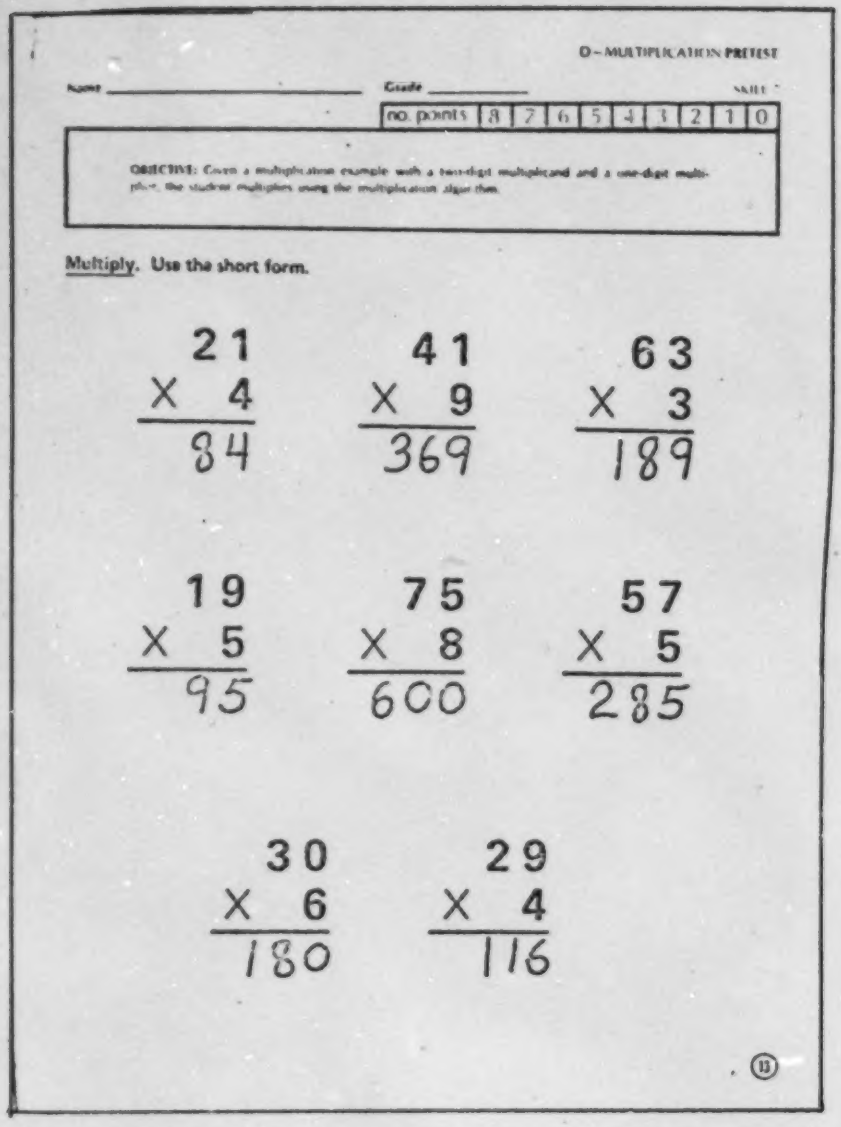
Multiplication Skills

Due to time constraints, analysis of systematic errors was completed for only two of the four multiplication skills for which analysis might have been carried out. The D-Multiplication-7 skill involves a one-digit multiplier and a two digit multiplicand. A three digit multiplicand with a one-digit multiplier is specified by D-Multiplication-9. It should be noted that the process of multiplication using partial products is taught at C-level. The D-level skill, however, requires the use of the usual "short" algorithm. For this reason a new "error" category, used partial products rather than the short algorithm, is included for the two multiplication skills. Figures 11 and 12, that follow, show the two pretests with the correct answer for each item. Tables 11 and 12 include descriptions of each systematic error noted and the number and precent of tests falling in each of the now seven defined categories. A total of 232 pretests were analyzed, 116 for each of the two skills. Most of the pretests for the two skills were completed by fourth graders.[42]

D-Multiplication-7. One or two of the eleven systematic errors described in Table 11 occurred on 20.8% of the 116 pretests. For this skill, the number of pretests showing two different systematic errors is not trivial. Of the eight pretests that were counted in two categories, the most frequent combination, five of the eight, included the first and second error pattern described in Table 11. In fact, all eight of the pretests that were tallied in two categories included the second systematic error described in Table 11. The number of pretests with systematic

---

[42]See Appendix B for the complete distribution of pretests over grade levels.



D – MULTIPLICATION PRETEST

Name _____ Grade _____ SCORE _____

no. points	8	7	6	5	4	3	2	1	0

OBJECTIVE: Given a multiplication example with a two-digit multiplicand and a one-digit multiplier, the student multiplies using the multiplication algorithm.

Multiply. Use the short form.

21 × 4 = 84	41 × 9 = 369	63 × 3 = 189
19 × 5 = 95	75 × 8 = 600	57 × 5 = 285
30 × 6 = 180	29 × 4 = 116	

13

Figure 11. D-Multiplication-7 Pretest with Answers

Table 11. D-Multiplication-7 Pretest Errors

Error				Frequency
Examples			Description	Number (percent)
$^{4}75$ x 8 = 280	$^{3}57$ x 5 = 155	$^{3}29$ x 4 = 66	Multiplication is performed in the ones' column. The one's digit of this product is recorded in the answer. The ten's digit of this product is multiplied by the ten's digit of the multiplicand and the product is placed in the answer.*	9 (7.8)
21 x 4 = 24	57 x 5 = 535	29 x 4 = 236	Only the one's digits are multiplied. This product is placed in the answer. The ten's digit of the multiplicand is then placed in the answer to the left of the ones' product.*	9 (7.8)
75 x 8 = 5640	57 x 5 = 2535	29 x 4 = 836	No renaming is performed. The product of the ones' digits is placed in the answer. The product of the one's digit of the multiplier and the ten's digit of the multiplicand is placed in the answer to the left of the ones' product.	4 (3.4)
$^{3}75$ x 8 = 806	$^{3}57$ x 5 = 405	$^{3}29$ x 4 = 206	Multiplication is performed in the ones' column. The one's digit of this product is placed in the answer. The ten's digit of the product is added to the ten's digit of the multiplicand. The resulting sum is multiplied by the one's digit of the multiplier and the product is placed in the answer.*	3 (2.6)
$^{40}75$ x 8 = 470	$^{30}57$ x 5 = 355	$^{30}29$ x 4 = 326	The ones' digits are multiplied correctly. The renamed ten's digit is written as a two digit multiple of ten and is added to the ten's digit of the multiplicand. The resulting sum is placed in the answer to the left of the one's column.	1 (0.8)

* Also noted by Linda S. Cox, (1974).

Table 11. D-Multiplication-7 Pretest Errors (con't)

Error: Examples			Error: Description	Frequency: Number (percent)
75 x 8 560	57 x 5 255	29 x 4 86	Multiplication is performed in the ones' column. The one's digit of this product is placed in the ones' column of the answer. (The ten's digit of the product is ignored.) The product of the multiplier and the ten's digit of the multiplicand is then placed in the answer to the left of the ones' column.	1 (0.8)
$^{4}75$ x 8 32560	$^{3}57$ x 5 15255	$^{3}29$ x 4 1286	Multiplication is performed in the ones' column. The ten's digit of this product is placed to the left of the multiplicand and is multiplied as if it were a hundred's digit. When multiplying the tens' column and "hundreds'" column, no renaming is performed. The partial products are placed in the answer.	1 (0.8)
$^{0}75$ x 8 04	$^{5}57$ x 5 253	$^{6}29$ x 4 123	Multiplication is performed in the ones' column. The ten's digit of this product is placed in ones' column of the answer. The one's digit of the first product is multiplied with the ten's digit of the multiplicand and the resulting product is placed in the answer to the left of the ones' column.	1 (0.8)
75 x 8 565	57 x 5 257	29 x 4 89	No multiplication is performed in the ones' column. The one's digit of the multiplicand is placed in the ones' column of the answer. The product of the multiplier and the ten's digit of the multiplicand is placed in the answer to the 'eft of the ones' column.	1 (0.8)

Table 11. D-Multiplication-7 Pretest Errors (con't)

Error				Frequency
Examples			Description	Number (percent)
$^{4}75$ x 8 = 320	$^{3}57$ x 5 = 155	$^{3}29$ x 4 = 126	Multiplication is performed in the ones' column. The one's digit of the product is placed in the ones' column of the answer. The ten's digit of this product is multiplied by the multiplier and the new product is placed in the answer to the left of the ones' column. (The ten's digit of the multiplicand is ignored.)	1 (0.8)
75 x 8 = 96	57 x 5 = 60	29 x 4 = 44	Each digit of the multiplicand is treated as a separate one's digit and is multiplied by the multiplier. The sum of the two products is placed in the answer. [75 x 8 = (5 x 8) + (7 x 8) = 40 + 56 = 96]	1 (0.8)
			Fact errors	1 (0.8)
			Three or more random errors	1 (0.8)
			Two errors	6 (5.2)
			One or no errors	66 (56.9)
			Did not attempt pretest	14 (12.1)
			Used partial products rather than the short algorithm	4 (3.4)

n = 116[1]

[1] The sum of the numbers is greater than 116, and the percents total more than 100, because one or more students were counted in more than one category.

errors far exceeds the number of pretests with three or more items incorrect due to fact errors or three or more random errors.

As noted in Appendix C, Cox's study includes a Multiplication, Level 3 skill that is comparable with D-Multiplication-7. Only three of the eleven systematic errors described in Table 11 are among the seven listed by Cox.[43] The systematic error listed first in Table 11, one of the two systematic errors found to be the most frequent in the present study, was the systematic error that Cox found to be most frequent.[44] Most of the systematic errors listed in Table 11 that Cox does not report are rather complex; some, such as the third systematic error described on page 52, are combinations of errors Cox described. The pre-instructional rate of 20.8% found in the present study is four times the 5% rate Cox reports for her post-instructional, normal, fourth grade sample.[45]

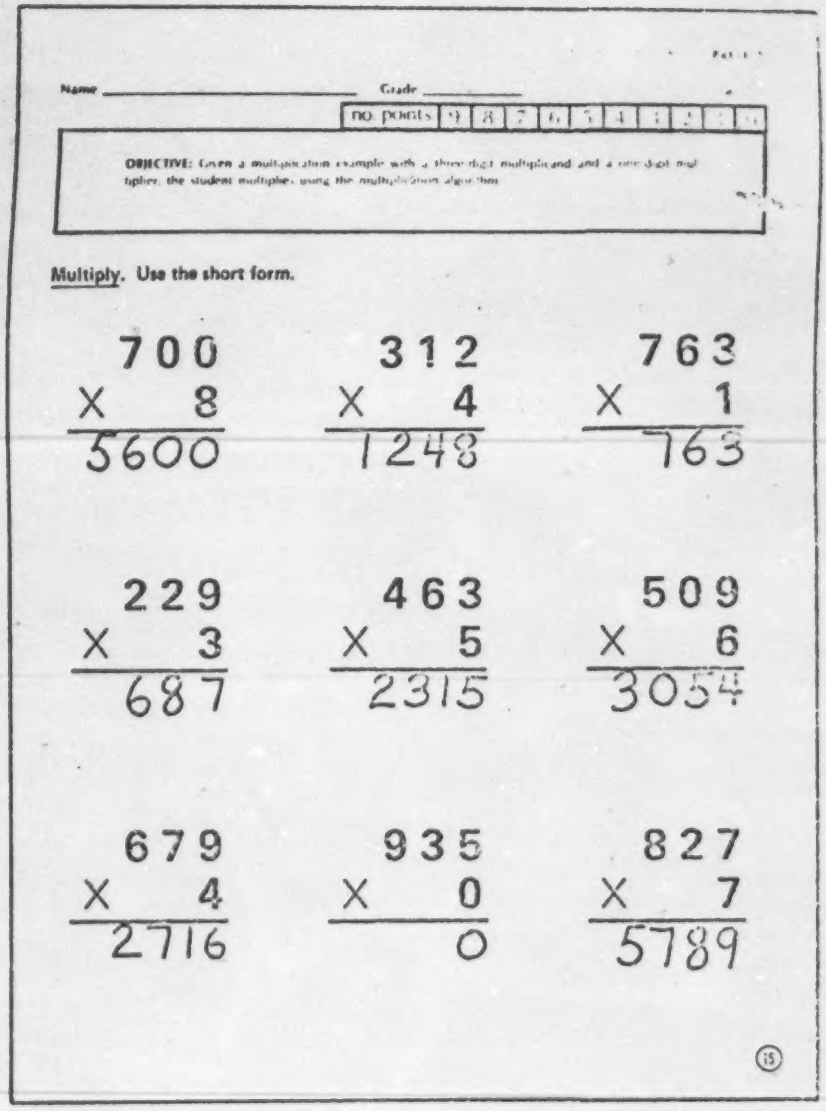
D-Multiplication-8. The D-Multiplication-8 pretest involves the multiplication of a three-digit number by a one-digit number. Again, the pretest directions ask for the "short form" of the algorithm. Many of the systematic errors, described in Table 12, are more complicated than either the actual short algorithm or the partial products algorithm and would hardly be characterized as "short." The grossest of estimates would show many answers to be ridiculous (e.g., 3 x 229 = 6,667; 229 x 3 = 47; 4 x 679 = 242,838). Although there are twelve systematic errors described in Table 12, some are slight variations of a basic patterns (as, for example, the first

---

[43]Cox, op. cit., pp. 81-82.

[44]Ibid., p. 81.

[45]Ibid., p. 78.



Name _____ Grade _____

no. points	9	8	7	6	5	4	3	2	1	0

OBJECTIVE: Given a multiplication example with a three-digit multiplicand and a one-digit multiplier, the student multiplies using the multiplication algorithm.

Multiply. Use the short form.

700 × 8 = 5600	312 × 4 = 1248	763 × 1 = 763
229 × 3 = 687	463 × 5 = 2315	509 × 6 = 3054
679 × 4 = 2716	935 × 0 = 0	827 × 7 = 5789

15

Figure 12. D-Multiplication-8 Pretest with Answers

Table 12. D-Multiplication-8 Pretest Errors

Error			Frequency
Examples		Description	Number (percent)
$\overset{4}{827} \times 7 = 889$	$\overset{1}{463} \times 5 = 465$	$\overset{23}{679} \times 4 = 1216$	Multiplication is performed in the ones' column. The one's digit of this product is recorded in the answer. The ten's digit of this product is multiplied by the ten's digit of the multiplicand and the one's digit of this product is placed in the answer. In general, the renamed digit of each product is multiplied by the next digit in the multiplicand. If there is no renamed digit, the digit in the multiplicand is placed in the answer.	8 (6.9)
$\overset{4\,4}{827} \times 7 = 8429$	$\overset{3\,1}{463} \times 5 = 3555$	$\overset{4\,3}{679} \times 4 = 4006$	Multiplication is performed in the ones' column. The one's digit of this product is placed in the answer. The ten's digit of this product is added to the ten's digit of the multiplicand. The resulting sum is multiplied by the multiplier. The process is repeated, i.e., the renamed digit is added to the hundred's digit of the multiplicand and the sum is multiplied by the multiplier.*	3 (2.6)
$\overset{1\,4}{827} \times 7 = 7562849$	$\overset{2\,3}{679} \times 4 = 8241286$	$\overset{2}{229} \times 3 = 6667$	Multiplication is performed correctly in the ones' column, but the digit to be renamed is recorded between the ten's and hundred's digit in the multiplicand. Thereafter this digit is treated as if it were a hundred's digit of the multiplicand. The ten's digit of the multiplicand is multiplied by the multiplier and the digit to be carried is placed to the left of the original hundred's digit of the multiplicand. Thereafter, this digit is treated as a ten thousand's digit of the multiplicand. In all subsequent multiplications no renaming is performed. Two digit partial products are placed in the answer.	2 (1.7)

* Also noted by Linda S. Cox, (1974).

Table 12. D-Multiplication-8 Pretest Errors (con't)

Error Examples			Error Description	Frequency Number (percent)
229 x 3 = 2227	463 x 5 = 4615	679 x 4 = 6736	Multiplication is performed in the ones' column and the product is placed in the answer without renaming. The remaining digits of the multiplicand are placed in the answer to the left of the first product. (No further multiplication is performed.)	2 (1.7)
312 x 4 = 008	229 x 3 = 047	763 x 1 = 003	Multiplication is performed in the ones' column. The one's digit of this product is recorded in the answer. The ten's digit of this product is multiplied by the ten's digit of the multiplicand. In general the renamed part of each product is multiplied by the next digit in the multiplicand. If there is no renamed digit, a zero is placed in the answer.	1 (0.8)
827 x 7 = 561449	229 x 3 = 6627	679 x 4 = 242836	No renaming is performed. Each partial product is placed in the answer.	1 (0.8)
827 x 7 = 1449	229 x 3 = 2627	679 x 4 = 62836	No renaming is performed. The partial products resulting from multiplication of the one's and ten's digit of the multiplicand are placed in the answer. No multiplication is performed with the hundred's digit; it is either ignored or placed in the answer.	1 (0.8)
827 x 7 = 6149	229 x 3 = 867	679 x 4 = 2986	Multiplication is performed in the ones' column. The one's digit of this product is recorded in the answer. The ten's digit of this product is placed to the left of the hundred's digit in the multiplicand. In general the renamed digit of each partial product is placed to the left of the hundred's digit in the multiplicand. The sum of these digits is then added to the partial product of the multiplier and the hundred's digit of the multiplicand.	1 (0.8)

Table 12. D-Multiplication-8 Pretest Errors (con't)

Error: Examples			Error: Description	Frequency: Number (percent)
$^{1}\,^{9}827 \times 7 = 6414$	$^{5}463 \times 5 = 031$	$^{2}\,^{4}679 \times 4 = 1243$	Multiplication is performed in the ones' column. The ten's digit of the product is placed in the ones' column of the answer. The one's digit of this product is multiplied by the ten's digit of the multiplicand. The "ten's" digit of this second product is placed in the answer. The "one's" digit of this second product is multiplied by the hundred's digit of the multiplicand. This third product is placed in the answer.	1 (0.8)
$312 \times 4 = 342$	$^{9}463 \times 5 = 1203$	$^{2}679 \times 4 = 1289$	No multiplication is performed in the ones' column. The one's digit of the multiplicand is placed in the answer. The product of the multiplier and the ten's digit of the multiplicand is calculated. The "one's" digit of this product is placed in the answer. If there is a renamed digit, its product with the hundred's digit of the multiplicand is placed in the answer. If there is no renamed digit, the hundred's digit of the multiplicand is placed in the answer.	1 (0.8)
$312 \times 4 = 1248$	$^{1}463 \times 5 = 2055$	$^{1}\,^{3}679 \times 4 = 426$	Multiplication is performed correctly in the one's column. If there is a digit to be renamed, it is multiplied by the multiplier and the "one's" digit of this product is placed in the tens' column of the answer. Again, if there is a renamed digit, it is multiplied by the multiplier and this product is placed in the hundreds' column of the answer. If at some step there is no renamed digit, multiplication is performed correctly.	1 (0.8)

Table 12. D-Multiplication-8 Pretest Errors (con't)

Error				Frequency
Examples			Description	Number (percent)
312 x 4 = 24	463 x 5 = 65	679 x 4 = 88	Each digit of the multiplicand is treated as a separate one's digit, and is multiplied by the multiplier. The sum of these products is placed in the answer. [312 x 4 = (3 + 1 + 2) 4 = 24]	1 (0.8)
			Fact errors	7 (6.0)
			Three or more random errors	13 (11.2)
			Two errors	11 (9.5)
			One or no errors	42 (36.2)
			Did not attempt pretest	20 (17.2)
			Used partial products instead of short algorithn	3 (2.6)

n = 116±

± The sum of the numbers is greater than 116, and the percents total more than 100, because one or more students were counted in more than one category.

and sixth patterns described). In all, 19% of the pretests contained systematic errors. One pretest showed two systematic errors, and two pretests showed a systematic error and included three items that were incorrect due to errors in basic facts. When compared with the eleven skills described previously, the pretests for the D-Multiplication-8 skill showed a comparatively high incidence of fact errors, 6%, and of three or more random errors, 13%.

Cox's Multiplication, Level 4 skill is the same as D-Multiplication-8. On her Multiplication, Level 4 tests Cox found six distinct systematic errors, only one of which was also found among the twelve systematic errors described in the present study.[46] The systematic error that Cox found to be most frequent among her handicapped population is a slight variation of the systematic error found to be most frequent in the present study.[47] The systematic error Cox found most prevalant among the normal population was the second most prevalant systematic error in the present study.[48] For this skill, the percent of papers with systematic errors from Cox's fourth grade normal sample was 6%.[49] Again this is about one-third the rate documented in the present study.

Discussion of Findings for Multiplication Skills. Since only two multiplication skills were analyzed, there are hardly sufficient data for describing trends or even for establishing the most frequent type of systematic error. However, a few observations can be made.

---

[46] Ibid., pp. 83-84.

[47] Ibid., p. 83.

[48] Ibid.

[49] Ibid., p. 78.

As was noted for the addition and subtraction skills, students who used a systematic error tended to do so repeatedly both on all the items on the pretest for a skill and on pretests for similar skills. For example, the student who on D-Multiplication-7 claimed that 29 x 4 = (2 + 9)4 = 44, claimed, on D-Multiplication-8, that 312 x 4 = (3 + 1 + 2)4 = 24.

With the exception of one additional place-value concept, the D-Multiplication-7 and 8 pretests involve the same component skills. Thus, the fact that the two skills showed the same types of systematic errors and a similar rate of systematic errors is probably to be expected.

Although the percent of three or more random errors was somewhat higher than on most previous skills, the percent of systematic errors is still greater than the percent of three or more random errors.

Disscussion and Implications

Table 13 shows the percent of pretests, for each skill, that falls into the six categories used throughout the study. The over-all percent of systematic errors for addition was 12%, for subtraction 40%, and for the two multiplication skills 19%. It is interesting to note that Cox also found the percent of multiplication and addition tests with systematic errors to be similar, while the percent of systematic errors on subtraction was considerably higher.[50]

Generally, the percent of students achieving mastery on the pretest for a skill can be used as an indication of the difficulty of a skill. It can be noted in Table 13 that pretests mastered by a high percent of students tended to have a lower rate of systematic errors than did skills mastered by a lower percent of students. Cox also noted this fact.[51]

Despite the fact that Cox used post-instructional work samples, the types of systematic errors described in the present study are also similar to those reported by Cox. In fact, on nine of the twelve skills analyzed in the present study, the systematic error that Cox reported as most frequent is the same as the systematic error found most frequent in the present study. Also, the types of problems that seemed to result in higher rates of systematic errors (e.g., subtraction with renaming across zeros) also resulted in systematic errors in the present study.

On all twelve skills the percent of pretests with systematic errors

---

[50] Ibid., p. 72.

[51] Ibid.

Distribution of Pretests by Skill and Grade Level

Area	Skill	Grade Level					Total by Skill	Total by Area
		2	3	4	5	6		
Addition Skills	C-Addition/Subtraction-3	36	39	5	1	0	71	
	C-Addition/Subtraction-4	34	29	5	1	0	69	
	C-Addition/Subtraction-7	35	29	5	1	0	70	
	D-Addition/Subtraction-3	0	16	50	20	14	100	
	D-Addition/Subtraction-4	0	16	48	21	14	99	
								409
Subtraction Skills	C-Addition/Subtraction-8	31	35	11	3	1	81	
	C-Addition/Subtraction-10	30	36	11	3	1	81	
	C-Addition/Subtraction-11	30	37	11	3	1	82	
	D-Addition/Subtraction-7	0	16	47	21	14	98	
	D-Addition/Subtraction-9	0	17	49	20	14	101*	
								443
Multiplication Skills	D-Multiplication-7	0	18	61	21	16	116	
	D-Multiplication-9	0	18	61	21	16	116	
								236
							Total Number of Pretests	1,088

*One pretest could not be identified with a grade level.
Therefore the total is one more than the sum of the given numbers.

was greater than the percent of pretests with fact errors in three or more items and also greater than the percent of pretests with three or more random errors. Furthermore, on only two skills, D-Addition/Subtraction-3 and 4, was the percent of pretests with two errors greater than the percent of pretests with systematic errors. Thus, many of the errors students made were systematic errors. Teachers, who may find individual interviews helpful but time consuming, may be able to diagnose the problems of many students from written work samples.

Only 32 of the 1,088 tests analyzed (2.9%) included three items that were incorrect due to fact errors. Of these 32, only eight pretests (0.7%) were tallied into both the fact error and systematic error categories. If poor computers are poor in basic facts and in algorithmic operations, either very few poor computers were encountered or the poor computers opted not to attempt the pretests.

Readers accustomed to employing diagnosis after instruction, or for remediation, may view the above analysis of pre-instructional systematic errors as interesting but useless. However, the tasks undertaken for the math diagnostic-prescriptive workscope assumed that diagnosis and prescription were integral parts of an instructional sequence and were not reserved exclusively for use in cases where students have been identified as in need of remedial instruction. (Certainly, within individualized programs like IPI, diagnosis of a pretest is carried out in order to prescribe relevant instruction, not for remediation.) The selection of relevant instruction is certainly crucial, although we would argue that it should always be the step that follows diagnosis. A study of advertisements for many of today's math programs indicate that the programs include pre-instructional diagnostic

tests. The fact that teachers do not always use information available on pretests in order to diagnose was noted earlier.[52] Diagnosis of systematic errors can probably be taught, but certainly examples must be available for use. While there is some repetition among the 82 systematic errors described in the present study, there are certainly many distinct patterns to which teachers who are responsible for prescribing instruction based on diagnostic test results could be exposed. Furthermore, since many of the systematic errors described in the study were the same as or similar to those reported by Cox in her study of post-instructional systematic errors, teachers who diagnose for the sake of remediation might also profit from acquaintance with the systematic errors described herein. Obviously, a teacher training unit would need to assume a format different from that of this research report. However, one potential use for the data in this report would be as part of a knowledge base in the development of such a teacher training package. Curriculum developers and teachers might also use the data in planning their instruction to be preventive instruction.

---

[52]See page 2 of this report.

APPENDIX A

IPI OBJECTIVE STATEMENTS FOR ANALYZED SKILLS AND THE RELATED TABLE IN COX'S STUDY[1]

[1] Linda S. Cox, Analysis, Classification, and Frequency of Systematic Error Computational Patterns in the Addition, Subtraction, Multiplication, and Division Vertical Algorithms for Grades 2-6 and Special Education Classes, Final report, U.S. Department of Health, Education & Welfare, National Institute of Education Contract #SE018001 (Kansas City: University of Kansas Medical Center, June, 1974.)

IPI Skill	IPI Objective	Related Table in Cox[2]	Notes
C-Addition/Subtraction-3	Given a two-digit addend and a one- or two-digit addend, the student adds using the standard algorithm. LIMIT: no regrouping; sums of 99	Table 11	
C-Addition/Subtraction-4	Given a two-digit addend and a one-digit addend, the student adds using the standard algorithm. LIMIT: sums of 99	Table 10	
C-Addition/Subtraction-7	Given two two-digit addends, the student adds using the standard algorithm. LIMIT: sums of 99	Table 12	The D-level skill includes checking, ignore the check.
D-Addition/Subtraction-3	Given a two-digit addend and a one- or two digit addend, the student adds using the standard algorithm and checks his answer using the commutative property.		
D-Addition/Subtraction-4	Given a three-digit addend and a one-, two-, or three-digit addend, the student adds using the standard algorithm. LIMIT: three-digit sums	Table 15	Categories may need to be added as Cox only considers a three-digit and two-digit addend.
C-Addition/Subtraction-8	Given a two-digit sum and a one- or two-digit addend, the student subtracts using the standard algorithm. LIMIT: no regrouping; sums of 99	Table 24	
C-Addition/Subtraction-10	Given a two-digit sum and any one-digit addend, the student subtracts using the standard algorithm.	Table 23	
C-Addition/Subtraction-11	Given a two-digit sum and a two-digit addend, the student subtracts using the standard algorithm.	Table 25	
D-Addition/Subtraction-7	Given a three- or four-digit sum and an addend, the student subtracts using the standard algorithm. LIMIT: four-digit addends; no regrouping from hundreds' or thousands' place	Tables 26 and 27	Categories may need to be revised as Cox's cases are more limited and IPI is missing some of the cases involving zeros.
D-Addition/Subtraction-9	Given a sum and an addend, the student subtracts using the standard algorithm. LIMIT: three- or four-digit sums; four-digit addends	Table 27	Categories may need to be revised as IPI is missing some of the cases involving zeros. Also Cox's first category is too broad.

[2]Ibid.

IPI Skills	IPI Objective	Related Level in Cox[3]	Notes
D-Multiplication-7	Given a multiplication example with a two-digit multiplicand and a one-digit multiplier, the student multiplies using the multiplication algorithm.	Table 33	
D-Multiplication-8	Given a multiplication example with a three-digit multiplicand and a one-digit multiplier, the student multiplies using the multiplication algorithm.	Table 34	

[3]Ibid.

APPENDIX B

DISTRIBUTION OF PRETESTS BY SKILL AND GRADE LEVEL

Table 13. Percent of Pretests Tallied in each Category by Skill

Category	Addition					Subtraction					Multiplication	
	C-Level			D-Level		C-Level			D-Level		D-Level	
	3	4	7	3	4	8	10	11	7	9	7	8
Systematic errors	8%	22%	31%	3%	2%	9%	57%	58%	31%	45%	21%	19%
Fact errors on 3 or more items	3	4	9	0	0	1	3	4	1	0	1	6
Three or more random errors	7	3	3	0	0	5	5	5	9	19	1	11
Two errors	7	13	10	4	4	6	6	2	6	4	5	10
Mastery (one or no errors)	76	54	41	82	92	76	21	18	48	19	57	36
Did not attempt pretest	0	4	10	11	2	2	9	16	5	12	12	20

APPENDIX C

DATA SHEET AND VOCABULARY LIST FOR SYSTEMATIC ERROR DESCRIPTIONS

Student Number _____

School Number _____

Circle Level	Circle Area	Skill Number _____	Did Not Attempt Pretest
C D E	A S M D		

Pretest

Procedural Error - Frequency _____

Fact Errors -

Posttest

Procedural Error - Frequency _____

Fact Errors -

Analysis By _____ Date _____

Vocabulary for Systematic Error Descriptions

ADDITION

37	(Top) Addend
+ 6	(Bottom) Addend
43	Answer

SUBTRACTION

47	Sum
- 66	Known Addend
41	Answer

MULTIPLICATION

32	Multiplicand
x 4	Multiplier
128	Answer

Number of digits: one-digit, two-digit, etc.

Places: ones' place, tens' place, etc.

Columns: ones' column, tens' column, etc.

"Regrouping," "carrying," "borrowing" or exchanging will usually all be renaming.

Sum, missing addend, and product will generally be reserved for the result of an intermediate step.